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Full Length Article

Free vibration analysis of pre-stressed FGM Timoshenko beams under large transverse deflection by a variational method

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ABSTRACT

A theoretical study on free vibration behavior of pre-stressed functionally graded material (FGM) beam is carried out. Power law variation of volume fraction along the thickness direction is considered. Geometric non-linearity is incorporated through von Kármán non-linear strain–displacement relationship. The governing equation for the static problem is obtained using minimum potential energy principle. The dynamic problem for the pre-stressed beam is formulated as an eigenvalue problem using Hamilton's principle. Three classical boundary conditions with immovable ends are considered for the present work, namely clamped–clamped, simply supported–simply supported and clamped–simply supported. Four different FGM beams, namely Stainless Steel–Silicon Nitride, Stainless Steel–Zirconia, Stainless Steel–Alumina and Titanium alloy–Zirconia, are considered for generation of results. Numerical results for non-dimensional frequency parameters of undeformed beam are presented. The results are presented in non-dimensional pressure-displacement plane for the static problem and in non-dimensional frequency-displacement plane for the dynamic problem. Comparative frequency-displacement plots are presented for different FGMs and also for different volume fraction indices.

1. Introduction

Functionally graded materials (FGMs) are inhomogeneous composites that have smooth and continuous variation of material properties in space. In most of the existing and potential future applications, FGM is considered mainly as a mixture of ceramic and metal in varying proportion. With the strength and toughness of metals, and the thermal and wear resistance of ceramics, FGM components possess good qualities of both the metals and ceramics. This makes it suitable for the FGM structures or components to be used in high temperature environment. FGM components are found in various applications, such as in aerospace, nuclear, automotive, civil, biomechanical, optical, electronic, mechanical, chemical and shipbuilding industries [1]. FGM components have applications in astronautic structures, such as rocket launch-pad, space vehicles [2], etc., because rocket launch-pad is subjected to tremendous thermal and mechanical loading, whereas, space vehicles are subjected to extreme thermal conditions. FGMs having excellent thermal and mechanical properties are suitable for such various astronautic structures. It is to be mentioned that the present work deals with FGM beams, which are often found in various structures in the fields of aerospace, mechanical, automotive, civil engineering, etc.

FGM beams are mainly designed for applications under thermal environment. But its behavior under mechanical loadings at ambient condition is also important in order to ascertain its performance when thermal loadings are absent. Knowledge of free vibration behavior of pre-stressed FGM beams under mechanical loading is important from design point of view. It is known that the amplitude of forced vibration becomes excessively large when the excitation frequency falls in the vicinity of the natural frequency of vibration of a loaded beam. To avoid such undesirable vibration levels, the natural frequency of vibration of the loaded beam must be known to the designer. Hence the present work is meant to investigate such dynamic behavior of FGM beams. The literature review of some related works by other notable researchers are given in the next few paragraphs.

Ke et al. [3] investigated the nonlinear vibration behavior of FGM beams based on Euler–Bernoulli beam theory and von Kármán geometric nonlinearity. Fallah and Aghdam [4,5] presented large amplitude free vibration analysis of FGM Euler–Bernoulli beams resting on nonlinear elastic foundation subjected to both mechanical and thermal loadings. Fu et al. [6] carried out nonlinear free vibration analysis of piezoelectric FGM beams under thermal

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environment employing Euler–Bernoulli beam theory. Lai et al. [7] obtained the accurate analytical solutions for large amplitude vibration of thin FGM beams using Euler–Bernoulli beam theory. Based on Euler–Bernoulli beam theory, Yaghoobi and Torabi [8] studied the nonlinear vibration behavior of geometrically imperfect FGM beams resting on nonlinear elastic foundation subjected to axial force. Hemmatnezhad et al. [9] studied the large-amplitude oscillations of FGM Timoshenko beams using finite element formulation. Rahimi et al. [10] performed free vibration analysis of FGM Timoshenko beams in the vicinity of a buckled equilibrium configuration.

Kapuria et al. [11] presented a theoretical finite element model for vibration analysis of layered FGM beams with experimental validation. Aydogdu and Taskin [12] studied free vibration behavior of simply supported FGM beams using different beam theories. Free vibration characteristics of simply supported FGM beams were investigated by Şimşek and Kocatürk [13] using Lagrange's equations under the assumptions of the Euler–Bernoulli beam theory. Thermo-mechanical vibration analysis of FGM beams resting on variable elastic foundation was carried out by Pradhan and Murmu [14]. Free vibration analysis of FGM beams based on a different first order shear deformation theory was carried out by Sina et al. [15]. Fundamental frequency analysis of FGM beams was carried out by Şimşek [16] using different higher-order beam theories. Giunta et al. [17] addressed free vibration behavior of functionally graded beams via several axiomatic refined theories. Using finite element method, Alshorbagy et al. [18] presented the free vibration characteristics of FGM beams with material graduation axially or transversally through the thickness based on the power law.

Free vibration characteristics of layered functionally graded beams were studied by Wattanasakulpong et al. [19] using Ritz method. Thai and Vo [20] investigated free vibration behavior of FGM beams based on various higher-order beam theories. Free vibration analysis of FGM beams for different boundary conditions was carried out by Pradhan and Chakraverty [21] using Euler–Bernoulli and Timoshenko beam theories. Free vibration behavior of axially loaded rectangular FGM beams was investigated by Nguyen et al. [22] based on the first-order shear deformation beam theory. The dynamic stiffness method was used by Su et al. [23] to investigate the free vibration behavior of FGM beams. Wattanasakulpong and Mao [24] investigated the dynamic response of Timoshenko FGM beams supported by various classical and non-classical boundary conditions.

Esfahani et al. [25] studied free vibration behavior of a thermally pre/post buckled FGM beam resting over a nonlinear hardening elastic foundation. Free vibration behavior of a thermo-electrically post-buckled rectangular FGM piezoelectric beams was studied by Komijani et al. [26]. Thermal buckling analysis of FGM beams with temperature-dependent material properties was carried out by Kiani and Eslami [27,28]. Esfahani et al. [29] carried out non-linear thermal stability analysis of temperature-dependent FGM beams resting on non-linear hardening elastic foundation. Thermo-electrical stability analysis of piezoelectric FGM beams had been carried out by Kiani et al. [30], Kargani et al. [31] and Komijani et al. [32], whereas thermal stability analysis of piezoelectric FGM beams was carried out by Kiani et al. [33].

The present work is based on Timoshenko beam theory, which considers uniform distribution of transverse shear stress across the beam thickness. It is worthwhile to mention some of the research works using higher shear deformation theories (HSDT) developed in the recent years for analysis of plate and beam structures. Tounsi et al. [34] carried out thermo-elastic bending analysis of functionally graded sandwich plates using a refined trigonometric shear deformation theory (RTSDT). The thermo-mechanical bending behavior of FGM plates resting on Winkler–Pasternak elastic foundations was studied by Bouderba et al. [35] using RTSDT. Buckling and free vibration behaviors of exponentially graded sandwich plates were investigated by Ait Amar et al. [36] using simple refined shear deformation theory. Static and dynamic analyses of FGM and sandwich plates had been carried out by Hebali et al. [37] and Mahi et al. [38] using new hyperbolic shear deformation theory. Using higher-order shear deformation theories, wave propagation analysis in porous FGM plates, and bending and vibration analysis of FGM plates, were carried out by Ait Yahia et al. [39] and Belabed et al. [40] respectively. Recently, Bourada et al. [41] developed a refined trigonometric higher-order beam theory to investigate static and dynamic behaviors of FGM beams. In that work, the authors have included stretching deformation effect along the thickness direction and eliminated the need of shear correction factor. Bousahla et al. [42] presented a new trigonometric higher-order theory for the static analysis of FGM plates employing the physical neutral surface concept. Hamidi et al. [43] presented a sinusoidal plate theory for the thermo-mechanical bending analysis of functionally graded sandwich plates. Bessaim et al. [44] developed a new higher-order shear and normal deformation theory for investigating the bending and free vibration behavior of sandwich plates with functionally graded isotropic face sheets. Thermo-elastic bending analysis of functionally graded sandwich plates was carried out by Bouchafa et al. [45] using a refined hyperbolic shear deformation theory. Houari et al. [46], using a new higher-order shear and normal deformation theory, simulated the thermo-elastic bending of FGM sandwich plates.

From the literature review presented, it is clear that an exhaustive study on free vibration behavior of transversely loaded beam for different FGM materials and different classical boundary conditions is scarce. Most of the published works are involved with either free vibration behavior of undeformed FGM beam or large amplitude free vibration behavior of FGM beam. Hence, in the present work, free vibration frequencies of FGM beam are computed for different pre-stressed configurations under uniform transverse pressure. Pre-stressed configurations are obtained through a geometrically non-linear static analysis. The linear vibration frequency of the pre-stressed beam, hereafter termed as loaded natural frequency, is then computed through an eigenvalue problem that includes the effect of pre-stressing using the displacement fields of the static problem. The effect of geometric non-linearity is included using von Kármán non-linear strain–displacement relationship. Timoshenko beam theory is used to consider the effects of shear deformation for the static problem and of rotary inertia for the subsequent dynamic problem. Suitable energy-based variational principles are used to derive the governing equations for both parts of the problem. Four different functionally graded materials and three different immovable classical boundary conditions are considered to show the pre-stressed dynamic behavior of beams.

2. Mathematical formulation

The present work aims at finding loaded natural frequency of pre-stressed FGM Timoshenko beam. A uniform rectangular beam with length L, height h and width b is considered. A beam with symbolic dimensions is shown in Fig. 1, where, x, y and z denote the coordinate axes along the length, width and thickness directions respectively. As mentioned earlier, two distinct but interrelated problems are formulated and solved to obtain the desired solution. The purpose of the first one, the static problem, is to obtain the pre-stressed configuration of the beam under the application of uniform transverse pressure. And the second problem, named as the dynamic problem, is utilized to obtain the loaded natural frequency of the deformed beam. It must be mentioned that the static configurations for different loadings are obtained through a geometrically non-linear analysis to address the large deflection effect.