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Full Length Article

Indirect fractional order pole assignment based adaptive control

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ABSTRACT

The design method of polynomial control laws by mean of pole placement are actually smart solutions to many industrial applications. This category of controllers is very popular in the industry; however most of their applications concern only problems with constant reference signals. In this paper, we propose an indirect adaptive controller by fractional order pole placement. The proposed control strategy is based on the self-tuning control structure and on-line estimation of the plant model parameters using the Recursive Least Squares (RLS) algorithm. To show the effectiveness of the proposed control scheme two simulation examples are presented. The first example is the control of a DC motor angular speed and the second one is the control of an air-lubricated capstan drive for precision positioning. Improvement in the system control dynamical behavior compared to classical control scheme has been shown for the two illustrative examples.

1. Introduction

Fractional adaptive control is a rapidly growing research topic [1]. Since the pioneering works of Vinagre et al. [2] and Ladaci and Charef [3] a decade ago, a great number of fractional adaptive control approaches have been developed. Some researchers have been interested in the Fractional Order Model Reference Adaptive Control (FOMRAC) [4–8]; others have investigated the fractional adaptive PID control domain [9,10]. Fractional order adaptive High-Gain control [11,12], fractional IMC-based adaptive control [13] and robust fractional order adaptive control [14,15] have also been introduced. Very recently fractional adaptive extremum seeking control has been investigated in Neçaibia et al. [16]. Because of the fractional, integral and derivative orders, the fractional order adaptive control laws offer new maneuver margins to the design engineers for control parameters tuning making it possible to improve significantly the controlled system's behavior and robustness. Applications of fractional order control are as various as robotics [3,9,14], autonomous guided vehicle (AGV) [17], hydraulic flight simulator [18], automatic voltage regulator [6], position servo systems [19], and renewable energy systems [20–22].

In this paper we propose a new fractional order adaptive control strategy based on the on-line indirect pole placement approach, by imposing fractional order poles while identifying the process model parameters in real time by mean of the least square estimation (LSE) method. The resulting fractional order control dynamics are implemented by making use of the so-called singularity function approximation method [23]. The rational function approximation obtained translates efficiently the controller synthesis to the classical integer order algebraic domain allowing easy computing of the control law from the general Diophantine equation. Two simulation examples are given to illustrate the enhanced performance obtained by the proposed indirect adaptive fractional pole placement control scheme and the usefulness of this real time control algorithm. The first example presents the application of this technique to the velocity control of a DC motor while as the second concerns the control of an air-lubricated capstan drive for precision positioning.

This paper is organized as follows: Section 2 presents some mathematical basics of the fractional calculus and the considered approximation approach for fractional order functions. Section 3 introduces the proposed indirect fractional order adaptive pole placement algorithm with the estimation method. Illustrative examples of control applications are presented in Section 4 to show the good performance of this control technique; finally some concluding remarks in Section 5 comment on this work.

2. Fractional order systems

The 19th century offered the major developments of the fractional-order derivative concept. Some recent reference books [24,25] provide a good source for the fractional calculus state of the art. However, application of fractional-order operators in dynamic feedback control systems is just a recent topic but it is gathering growing interest [26–29].

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2.1. Basic definitions

Fractional order integrals and derivatives are generalizations of the classical (integer order) ones. Non-integer order fundamental operators are commonly represented as ${}_aD_t^\mu$ where a and t are the limits and $\mu(\mu \in \Re)$ the order of the operation. Researches in such an ambiguous topic leaded mathematicians to many (different) definitions of this operator [25].

One of the fractional integro-differential operator definitions that have most popularity is the Riemann–Liouville (RL) definition:

$$ {}_aD_t^\mu f(t) = \frac{1}{\Gamma(1-\mu)} \frac{d^n}{dt^n} \int_a^t (t-\xi)^{-\mu} f(\xi) d(\xi) \tag{1}$$

where $\Gamma(.)$ is the Euler's Gamma function, $(a,t) \in \mathbb{R}^2$ with $a < t$ and n an integer.

The Laplace transform of the Riemann–Liouville fractional operator (1) under null initial conditions for the order μ, $(0 < \mu < 1)$ is given by

$$L\{{}_aD_t^{\pm\mu} f(t); s\} = s^{\pm\mu} F(s) \tag{2}$$

A single input single output (SISO) fractional order system can be represented by the following transfer function,

$$F(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots + a_0 s^{\alpha_0}} \tag{3}$$

where α_i and β_j are real numbers such that,

$$\begin{cases} 0 \le \alpha_0 < \alpha_1 < \ldots < \alpha_n \\ 0 \le \beta_0 < \beta_1 < \ldots < \beta_m \end{cases}$$

and s is Laplace operator.

2.2. Approximated fractional second-order transfer function

The main problem with fractional order control design is that the resulting functions are of infinite dimension, whereas the implementation of such controllers needs to be realized by finite dimensional approximated linear filters.

This implies for the present work the need of an approximation method in order to replace the resulting fractional order functions by quasi-equivalent rational transfer functions in order to assign fractional order dynamics to the controlled system closed loop. In order to achieve this goal, we shall use the simple but popular *singularity function method* for approximation in the frequency domain [23,30].

For the need of the case study below, let us focus on the fractional standard second-order transfer function given by:

$$G_f(s) = \frac{1}{\left(\frac{s^2}{\omega_n^2} + 2\xi \frac{s}{\omega_n} + 1\right)^\beta} \tag{4}$$

where ξ is the damping factor, ω_n the proper pulse and $0 < \beta < 1$.

This function is usually employed as reference model in control systems design because of the well known properties of the system (4) in terms of the fractional order β and the damping factor effects on time response [31].

The singularity function method makes it possible to approximate the fractional order transfer function (4) by a quotient of polynomials in s. Two cases are distinguished

- **Case** $0 < \beta < 0.5$:

We can express the function (4) as follows:

$$G_e(s) = \frac{\left(\frac{s}{\omega_n} + 1\right)\left(\frac{s}{\omega_n + 1}\right)^\eta}{\left(\frac{s^2}{\omega_n^2} + 2\alpha \frac{s}{\omega_n} + 1\right)} \tag{5}$$

with $\alpha = \xi^\beta$ and $\eta = 1 - 2\beta$, which can also be approximated by the function,

$$G_e(s) \approx \frac{\left(\frac{s}{\omega_n} + 1\right)}{\left(\frac{s^2}{\omega_n^2} + 2\alpha \frac{s}{\omega_n} + 1\right)} \frac{\prod_{i=1}^{N-1}\left(1 + \frac{s}{z_i}\right)}{\prod_{i=1}^{N}\left(1 + \frac{s}{p_i}\right)} \tag{6}$$

The singularities (poles p_i and zeros z_i) are given by the following formulas:

$$p_i = (ab)^{i-1} a z_1 \quad i = 1, 2, 3, \ldots, \tag{7}$$

$$z_i = (ab)^{i-1} z_1 \quad i = 2, 3, \ldots, N-1 \tag{8}$$

with,

$$z_1 = \omega_n \sqrt{b} \tag{9a}$$

$$a = 10^{\frac{\varepsilon_p}{10(1-\eta)}} \tag{9b}$$

$$b = 10^{\frac{\varepsilon_p}{10\eta}} \tag{9c}$$

$$\eta = \frac{\log(a)}{\log(ab)} \tag{9d}$$

ε_p is the tolerated error in dB. The approximation order N is calculated by fixing the working frequency bandwidth, specified by ω_{max} such that: $p_{N-1} < \omega_{max} < p_N$, which leads to the following value:

$$N = \text{Integer part of} \left[\frac{\log\left(\frac{\omega_{max}}{p_1}\right)}{\log(ab)} + 1 \right] + 1 \tag{10}$$

$G_e(s)$ can then be rewritten under the form of a parametric function of order $N + 2$ as in Equation (11).

$$G_e(s) = \frac{b_{m_0} s^N + b_{m_1} s^{N-1} + \ldots + b_{m_N}}{s^{N+2} + a_{m_1} s^{N+1} + \ldots + a_{m_{N+2}}} \tag{11}$$

The coefficients a_{m_i} and b_{m_i} are computed from the singularities p_i, z_i as well as α and ω_n.

- **Case** $0.5 < \beta < 1$:

The fractional transfer function is rearranged as follows:

$$G_e(s) = \frac{\left(\frac{s}{\omega_n} + 1\right)}{\left(\frac{s^2}{\omega_n^2} + 2\alpha \frac{s}{\omega_n} + 1\right)\left(\frac{s}{\omega_n + 1}\right)^\eta} \tag{12}$$

where $\alpha = \xi^\beta$ and $\eta = 2\beta - 1$, developed as mentioned above with the following values of poles and zeros:

$$p_i = (ab)^{i-1} p_1 \quad i = 1, 2, 3, \ldots, N \tag{13}$$

$$z_i = (ab)^{i-1} a p_1 \quad i = 2, 3, \ldots, N-1 \tag{14}$$