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Continuous Optimization

A nonlinear interval number programming method for uncertain optimization problems

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Abstract

In this paper, a method is suggested to solve the nonlinear interval number programming problem with uncertain coefficients both in nonlinear objective function and nonlinear constraints. Based on an order relation of interval number, the uncertain objective function is transformed into two deterministic objective functions, in which the robustness of design is considered. Through a modified possibility degree, the uncertain inequality and equality constraints are changed to deterministic inequality constraints. The two objective functions are converted into a single-objective problem through the linear combination method, and the deterministic inequality constraints are treated with the penalty function method. The intergeneration projection genetic algorithm is employed to solve the finally obtained deterministic and non-constraint optimization problem. Two numerical examples are investigated to demonstrate the effectiveness of the present method.

Keywords: Uncertain optimization; Nonlinear programming; Interval number; Genetic algorithm

1. Introduction

In traditional mathematical programming, the coefficients of the problems are always treated as deterministic values. However uncertainty always exits in practical engineering problems. In order to deal with the uncertain optimization problems, fuzzy and stochastic approaches are commonly used to describe the imprecise characteristics. In stochastic programming (e.g. Charnes and Cooper, 1959; Kall, 1982; Liu et al., 2003; Cho, 2005) the uncertain coefficients are regarded as random variables and their probability distributions are assumed to be known. In fuzzy programming (e.g. Slowinski, 1986; Delgado et al., 1989; Luhandjula, 1989; Liu and Iwamura, 2001) the constraints and objective function are viewed as fuzzy sets and their membership functions also need to be known. In these two kinds of approaches, the membership functions and probability

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distributions play important roles. However, it is sometimes difficult to specify an appropriate membership function or accurate probability distribution in an uncertain environment (Sengupta et al., 2001).

In recent years, the interval analysis method was developed to model the uncertainty in uncertain optimization problems, in which the bounds of the uncertain coefficients are only required, not necessarily knowing the probability distributions or membership functions. Tanaka et al. (1984) and Rommelfanger (1989) discussed the linear programming problem with interval coefficients in objective function. Chanas and Kuchta (1996a,b) suggested an approach based on an order relation of interval number to convert the linear optimization problem with uncertainty into a deterministic optimization problem. Tong (1994) investigated the problems in which the coefficients of the objective function and the constraints are all interval numbers. He obtained the possible interval of the solution by taking the maximum value range and minimum value range inequalities as constraint conditions. Liu and Da (1999) proposed an interval number optimization method based on a fuzzy constraint satisfactory degree to deal with the linear problems. Sengupta et al. (2001) studied the linear interval number programming problems in which the coefficients of the objective function and inequality constraints are all interval numbers. They proposed the concept of ''acceptability index'' and gave one solution for the uncertain linear programming. Zhang et al. (1999) assumed interval numbers as random variables with uniform distributions and constructed a possibility degree to solve the multi-criteria decision problem. The above methods point out a fine way for the uncertain optimization. However, only linear programming problems are investigated. For most of the engineering problems, the objective function and constraints are nonlinear, and they are always obtained through numerical algorithms such as finite element method (FEM) instead of the explicit expressions. Furthermore, only the linear inequality constraints are studied and they have not proposed an approach to deal with the uncertain equality constraints. The reference (Ma, 2002), on the best knowledge of the authors, seems the first and only publication on nonlinear interval number programming (NINP). In this reference, a deterministic optimization method is used to obtain the interval of the nonlinear objective function, and a three-objective optimization problem is formulated. However, only the uncertain objective function is considered, and no approach is proposed to deal with the nonlinear constraints with uncertainty. As a result, an effective method still have not been developed to deal with the general NINP problem in which there exit not only uncertain nonlinear objective function but also uncertain nonlinear constraints, so far.

In this paper, an NINP method is firstly suggested to deal with the general nonlinear optimization problems. An order relation of interval number is used to transform the uncertain single-objective optimization into a deterministic two-objective optimization, which considers the midpoint and radius of the uncertain objective function simultaneously. A modified possibility degree of interval number based on the probability method is suggested to deal with the uncertain inequality constraints. The uncertain equality constraint is firstly investigated, and it is solved by being transformed into two uncertain inequality constraints. For each specific decision vector, two deterministic optimization processes are performed to obtain the interval of the objective function or constraint. An unconstraint and single-objective optimization problem is finally formulated through the linear combination method and the penalty function method. The intergeneration projection genetic algorithm (IP-GA) with fine global convergence performance is employed as optimization solver. A benchmark test is presented to demonstrate the effectiveness of the present method, and then this method is applied to a practical engineering problem, namely the locators' optimization of an automobile welding fixture with uncertain load and material property.

2. Statement of the problem

A general nonlinear interval number programming (NINP) problem with uncertain interval coefficients in both of the objective function and constraints can be given as follows:

$$\begin{aligned} \min \quad & f(\mathbf{X},\mathbf{U}) \\ \text{s.t.} \quad & g_i(\mathbf{X},\mathbf{U}) \geqslant (=,\leqslant)[v_i^{\mathrm{L}}, v_i^{\mathrm{R}}], \quad i=1,\ldots,l, \\ & \mathbf{X}\in\Omega^n, \quad \mathbf{U}=[\mathbf{U}^{\mathrm{L}},\mathbf{U}^{\mathrm{R}}], \quad U_i=[U_i^{\mathrm{L}},U_i^{\mathrm{R}}], \;\; i=1,2,\ldots,q, \end{aligned} \tag{1}$$