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A single period inventory model with imperfect production and stochastic demand under chance and imprecise constraints

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Abstract

This paper develops a mathematical model for a single period multi-product manufacturing system of stochastically imperfect items with continuous stochastic demand under budget and shortage constraints. After calculating expected profit in general form in terms of density functions of the demand and percentage of imperfectness, particular expressions for those density functions are considered. Here the constraints are of three types: (i) both are stochastic, (ii) one stochastic and other one imprecise (fuzzy) and (iii) both imprecise. The stochastic constraints have been represented by chance constraints and fuzzy constraints in the form of possibility/necessity constraints. Stochastic and fuzzy constraints are transformed to equivalent deterministic ones using 'here and now' approach and fuzzy relations respectively. The deterministic problems are solved using a non-linear optimization technique-Generalized Reduced Gradient Method. The model is illustrated through numerical examples. Sensitivity analyses on profit functions due to different permitted 'aspiration' and 'confidence' levels are presented.

Keywords: Inventory; Stochastic demand; Imperfect production; Chance constraint; Possibility/Necessity constraints

1. Introduction

In most of the classical economic order quantity (EOQ) inventory models developed since 1915, the demand is considered to be deterministic - constant or time-dependent or stock dependent. In some realistic situations like newsboy problem etc., demand is uncertain in stochastic sense.

Hadley and Whitin [20] first extended the classical EOQ inventory model to the stochastic model. Traditional stochastic models have been discussed primarily with the demand and lead time uncertainties [cf. Nahmias [37] and Silver [42]]. Contini [7] developed an algorithm for stochastic goal programming in which the random variables are normally distributed with known means and variances. In this model the stochastic problem has been transformed into an equivalent deterministic quadratic programming problem, where the

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objective functions consisted of maximizing the probability of a vector of goals lying in the confidence region of predefined size. Fitzsimmoms and Sullivan [15] developed an algorithm using probabilistic goals based on the concept of chance constraints due to Charnes and Cooper [5] where the goals can be stated in terms of probability of satisfying the aspiration levels. Teghem et al. [43] and Leclercq [30] presented interactive methods in stochastic programming. Shah [41] discussed a probabilistic time scheduling model for an exponential decaying inventory when delays in payment are permissible. Ben-Daya and Raouf [2], Kalpakam and Sapan [25,26] studied perishable inventory models with stochastic lead-time. Karmarkar [27] derived a lot size model with lead time and established the relationships between lot sizes and lead times for batch manufacturing shops with queues. The models discussed above are EOQ models that deal with instantaneous inventory replenishment policy only.

An economic production quantity (EPQ) model deals with an inventory -cum production system in which procurement of inventory occurs through production within the cycle itself. Till date several research papers have been developed on EPQ models having constant or dynamic or stock dependent demand with/without shortages. In some models unit production costs are taken as functions of production rate. Khouja [28], Khouja and Mehrej [29] solved EPQ model considering constant demand and no shortage, taking the production rate as a decision variable. Hong et al. [21] developed an EPQ model considering linearly time varying demand and constant production rate. Goswami and Chaudhuri [17] developed such a model considering shortages. Zhou [49,50] considered EPQ models over a finite time horizon with a linear trend in demand and shortages. He considered different production rate in different production cycle to fulfill the increasing demand. Giri and Chaudhuri [16] discussed a production-inventory model considering linearly time dependent demand and production rate dependent unit production cost with fully backlogged shortages. Aliyu et.al [1] developed an EPQ system with shortages and back order. Sana et al [40] developed an EPQ model for deteriorating items with trended demand and shortages. Maity and Maiti [34] extended the research in considering the EPQ model with inventory dependent demand under inflation and discounting.

The models discussed above did not consider the defectiveness in the produced quantities. In reality all the produced units can not be of good quality in a production system. Defective items as a result of imperfect production process were initially considered by Proteous [38] and later by several researchers such as Goyal and Cardenas-Barron [18], Ben-Daya [3], Goyal et al. [19]. Salama and Jaber [39] considered imperfect quality items in stochastic environment.

In general, the EPQ models are formulated under crisp resource constraints. In real life, it may not be so. For example, at the beginning of a business, it may be launched with some capital. But during the period of business, it may happen that to meet the unexpected increased demand, the production rate may have to be stepped up and in doing so, the organization would have to invest some more capital. This augmented amount is normally fuzzy in nature for a new company, for which past data are not available. Regarding an old company, it is possible to have the past data for the variation in the budget and the said may be represented by a probability distribution. Hence the resource constraints become stochastic or imprecise in nature. During the last few years, there are some EPQ models [cf. Chang [4], Chen and Hsieh [6], Hsieh [23], Hsieh and Chen [22], Lee and Yao [31], Tsourveloudis et al. [44], Yao and Wu [47]] formulated in fuzzy environment in the literature. Earlier investigations normally took one parameter or a resource as fuzzy and solved using fuzzy set theory and extension principle. They did not consider fuzzy and stochastic parameters and/or both fuzzy and stochastic constraints in a single model. In these models, resource constraints were not imposed in possibility/necessity sense. Such a complex model has been investigated in this paper.

The chance constrained programming (CCP) technique is one which is used to solve problems involving chance constraints i.e., constraints having random parameters. The CCP was originally developed by Charnes and Cooper [5] and has, in recent years, been extended in several directions for various applications. Liu and Iwamura [32] solved chance constraint programming with fuzzy parameters.

Analogous to chance constraint programming with stochastic parameters, in a fuzzy environment, it is assumed that some constraints with a least possibility are satisfied. Again some constraints may be satisfied with some predefined necessity also [cf. Dubois and Prade [10,11]]. These possibility and necessity resource constraints may be imposed as per the demand of the situation. Zadeh [48], Dubois and Prade [8,9,13,14], Wang and Hwang [46] introduced the necessity and possibility constraints which are very relevant to the real life decision making problems and presented the process of defuzzification for these constraints. Liu and