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Single item lot-sizing problem for a warm/cold process with immediate lost sales

Emre Berk *, Ayhan Özgür Toy [1], Öncü Hazır [1]

Faculty of Business Administration, Bilkent University, 06800 Ankara, Turkey

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Abstract

We consider the dynamic lot-sizing problem with finite capacity and possible lost sales for a process that could be kept *warm* at a unit variable cost for the next period $t+1$ only if more than a threshold value Q_t has been produced and would be *cold*, otherwise. Production with a cold process incurs a fixed positive setup cost, K_t and setup time, S_t, which may be positive. Setup costs and times for a warm process are negligible. We develop a dynamic programming formulation of the problem, establish theoretical results on the structure of the optimal production plan in the presence of zero and positive setup times with Wagner–Whitin-type cost structures. We also show that the solution to the dynamic lot-sizing problem with lost sales are generated from the full commitment production series improved via lost sales decisions in the presence of a warm/cold process.

Keywords: Dynamic lot sizing; Warm/cold process; Lost sales; Setup times; Setup costs; Scheduling

1. Introduction

In this paper, we consider the dynamic lot-sizing problem with finite capacity and lost sales for a process that can be kept *warm* onto the next period at some variable cost provided that the production in the current period exceeds some given threshold. The process industries such as glass, steel and ceramic are the best examples where the physical nature of the production processes dictates that the processes be literally kept warm in certain periods to avoid expensive shutdown/startups. A striking instance comes from the glass industry. In some periods, production of glass is continued to avoid the substantial shutdown/startup costs and the produced glass is *deliberately broken* on the production line and *fed back* into the furnace. Similar practices are employed in foundries; ceramic and brick ovens are also kept warm sometimes even though no further production is done in the current period to avoid costly cooling-and-reheating procedures. Furthermore, Robinson and Sahin (2001) cite specific examples in food and petrochemical industries where certain cleanup and inspection

* Corresponding author. Fax: +90 312 2664958.
E-mail addresses: eberk@bilkent.edu.tr (E. Berk), aotoy@bilkent.edu.tr (A.Ö. Toy), oncuh@bilkent.edu.tr (Ö. Hazır).
[1] Fax: +90 312 2664958.

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operations can be avoided in the next period if the quantity produced in the current period exceeds a certain threshold so that the current production continues onto the next period. This may be done through either overtime or undertime. The treatment of the overtime option is outside the scope of our analysis; however, deliberate undertime practices can be studied within our context of warm/cold processes. With undertime, processes can be kept warm in environments with variable production rates by reducing the "nominal" or "calibrated" rate within a prespecified range (*e.g.*, Silver, 1990; Moon et al., 1991; Gallego, 1993). As an illustration, suppose that the process is capable of producing at most R units per time period at a nominal production rate and that its production rate can be reduced so that, within the same time period, the process can produce Q ($<R$) units at the slowest rate. Thus, it is possible to keep this process warm by having it operate at rates lower than nominal so long as the quantity to be produced is between Q and R. Such variable production rates are quite common in both process and discrete item manufacturing industries – feeder mechanisms can be adjusted so as to set almost any pace to a line; some chemical operations such as electroplating and fermentation can be decelerated deliberately (within certain bounds), and, manual operations can be slowed down by inserting idle times between units. Depending on the nature of the operations involved, the reduction in production rate can be obtained at either zero or positive additional cost. This additional variable cost is then the variable cost of keeping the process warm onto the next period. Aside from merely economic calculations, non-economic considerations such as safety of mounted tools and fixtures left idle on the machinery, impact on worker morale of engaging them in non-productive activities for a longer duration, impact of learning/forgetting phenomena on subsequent runs, *etc.* may also result in a managerial decision on a warm process threshold. The dynamic lot-sizing problem in the presence of production quantity-dependent warm/cold processes is a rather common problem; however, it has only recently been studied in Toy and Berk (2006). In their work, no shortages are allowed. In this work, we allow for possible lost sales. Before discussing the particulars of our work, we briefly review related works in the vast dynamic lot-sizing literature below.

The first formulation of the dynamic lot-sizing problem is independently by Wagner and Whitin (1958) and Manne (1958). The so-called Wagner–Whitin–Manne problem assumes a single item, uncapacitated production, no shortages and zero setup times. Wagner and Whitin (1958) provided a dynamic programming solution algorithm and structural results on the optimal solution of the classical problem which enable the construction of a forward solution algorithm. Recently, Aksen et al. (2003) extended the results of Wagner and Whitin to the case of immediate lost sales and showed that a forward polynomial algorithm is possible in this case, as well. When production capacity in a period is limited, the problem becomes the so-called capacitated lot-sizing problem (CLSP). The single-item CLSP has been shown to be *NP-hard* by Florian and Klein (1971). Certain special cases of the CLSP of horizon length T have been shown to be solvable in polynomial time (Florian and Klein, 1971; Van Hoesel and Wagelmans, 1996; Bitran and Yanasse, 1982; Chung and Lin, 1988; Sandbothe and Thompson, 1990; Liu et al., 2004). For the CLSP for multi-item with positive setup times, we refer to a recent work, Gupta and Magnusson (2005). For a comprehensive review of the existing literature on the extension of the classical problem, we refer the reader to Wolsey (1995), Karimi et al. (2003) and Brahimi et al. (2006). Reformulations and algorithms for capacitated and uncapacitated lot-sizing problems are provided in Aggarwal and Park (1993) and Pochet and Wolsey (1995).

All of these works are different from the setting considered herein in that the production process is always cold, that is, it needs to be readied at positive cost for production in a particular period. The works that are close to our warm process setting are those in which it is possible to reserve a certain period for future production. This setting also occurs as a subproblem of the multi-item capacitated lot-sizing problem with the Lagrangian multipliers as the reservation costs for each of the periods and has been studied by Karmarkar et al. (1987), Eppen and Martin (1987), Hindi (1995a,b), and Agra and Constantino (1999). Although the models on lot sizing with reservation options employ the notion of a warm process, they do not consider a lower bound on the quantity produced for keeping the process warm onto the next period. We should also mention the related vast literature on the multi-item capacitated lot-sizing problems with sequence-dependent setups which consider warm processes but assumes only warm process thresholds of zero production. (See Allahverdi et al., 1999, this issue, for an extensive review on this subject.) Despite similarities, the above cited works are not readily applicable to our setting since we consider an explicit positive warm process threshold. Another body of work that uses the notion of warm processes is the discrete lot-sizing and scheduling problem (DLSP) literature (*e.g.* Fleischmann, 1990; Bruggemann and Jahnke, 2000; Loparic et al., 2003). This group of work differs from ours