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Fuzzy inference to assess manufacturing process capability with imprecise data

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Abstract

Process capability indices provide numerical measures on whether a process conforms to the defined manufacturing capability prerequisite. These have been successfully applied by companies to compete with and to lead high-profit markets by evaluating the quality and productivity performance. The loss-based process capability index C_{pm}, sometimes called the Taguchi index, was proposed to measure process capability, wherein the output process measurements are precise. In the present study, we develop a realistic approach that allows the consideration of imprecise output data resulting from the measurements of the products quality. A general method combining the vector of fuzzy numbers to produce the membership function of fuzzy estimator of Taguchi index is introduced for further testing process capability. With the sampling distribution for the precise estimation of C_{pm}, two useful fuzzy inference criteria, the critical value and the fuzzy *P-value*, are proposed to assess the manufacturing process capability based on C_{pm}. The presented methodology takes into the consideration of a certain degree of imprecision on the sample data and leads to the three-decision rule with the four quadrants decision-making plot. The fuzzy inference procedure presented to assess process capability is a natural generalization of the traditional test, when the data are precise the proposed test is reduced to a classical test with a binary decision.

Keywords: Process yield; Fuzzy sets; Fuzzy hypothesis testing; Critical value; Fuzzy *P-value*; Three-decision rule

1. Introduction

The loss-based process capability index C_{pm}, sometimes called the Taguchi index, proposed separately by Chan et al. (1988) and Hsiang and Taguchi (1985), was proposed to measure process capability. The index C_{pm} incorporates with the variation of production items with respect to the target value and the specification limits preset in the factory. It measures the ability of the process and reflects the density of the data about the target

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Nomenclature

C	process capability requirement
c_0	critical value
c^*	value of $\hat{C}_{pm}$ calculated from the sample data
C_{pm}	Taguchi index
$\hat{C}_{pm}, \hat{C}'_{pm}$	point estimators of C_{pm}
$\tilde{\hat{C}}_{pm}$	fuzzy estimator of C_{pm}
$\hat{c}_{pm}$	point estimate of $\hat{C}_{pm}$
d	half of the length of the specification interval
$\tilde{\mathbf{E}}$	q-dimensional fuzzy subsets vector $\tilde{E}_1, \tilde{E}_2, \ldots, \tilde{E}_q$
$\mathbf{e}$	q estimates $e_1, e_2, \ldots, e_q$ results in q fuzzy subsets $\tilde{E}_1, \tilde{E}_2, \ldots, \tilde{E}_q$
$f_{\hat{C}_{pm}}(x)$	probability density function (PDF) of $\hat{C}_{pm}$
$F_{\hat{C}_{pm}}(x)$	cumulative distributed function (CDF) of $\hat{C}_{pm}$
$G(\cdot)$	CDF of the χ^2 distribution with degree of freedom $n-1$, χ^2_{n-1}
$I_A(\cdot)$	indicator function of a classical set A
LSL	lower specification limit
USL	upper specification limit
T	target value
l	lower bound
$l_{\tilde{Z}}(\beta) = \inf_{z \in \tilde{Z}[\beta]}(z)$	lower bound of the β-*cut* of the fuzzy number $\tilde{Z}$
$l_{\tilde{\hat{C}}_{pm}}(\beta)$	lower bound of the β-*cut* of the fuzzy number $\tilde{\hat{C}}_{pm}$
$l_{\tilde{P}}(\beta)$	lower bound of the β-*cut* of the fuzzy number $\tilde{P}$
ψ	upper bound
$\psi_{\tilde{Z}}(\beta) = \sup_{z \in \tilde{Z}[\beta]}(z)$	upper bound of the β-*cut* of the fuzzy number $\tilde{Z}$
$\psi_{\tilde{\hat{C}}_{pm}}(\beta)$	upper bound of the β-*cut* of the fuzzy number $\tilde{\hat{C}}_{pm}$
$\psi_{\tilde{P}}(\beta)$	upper bound of the β-*cut* of the fuzzy number $\tilde{P}$
MLE	maximum likelihood estimator
m	midpoint of the specification limits
n	sample size
NC	fraction of nonconformities
P-value	actual risk of misjudging an incapable process as a capable one
PPM	parts per million
S^2, S_n^2	sample variances
s^2, s_n^2	point estimates of sample variances
$\tilde{S}^2, \tilde{S}_n^2$	fuzzy estimators of sample variances
UMVUE	uniformly minimum variance unbiased estimator
X	random variable with the normal distribution
x	value of the random variable X
$\overline{X}$	sample mean
$\bar{x}$	point estimate of the sample mean
$\tilde{\overline{X}}$	fuzzy estimator of the sample mean $\overline{X}$
Y	random variable
y	value of the random variable Y
$\tilde{Z}[\beta] = [l_{\tilde{Z}}(\beta), \psi_{\tilde{Z}}(\beta)]$	β-*cut* of the fuzzy number $\tilde{Z}$
$\tilde{\hat{C}}_{pm}[\beta] = [l_{\tilde{\hat{C}}_{pm}}(\beta), \psi_{\tilde{\hat{C}}_{pm}}(\beta)]$	β-*cut* of the fuzzy number $\tilde{\hat{C}}_{pm}$
$\tilde{P}[\beta] = [l_{\tilde{P}}(\beta), \psi_{\tilde{P}}(\beta)]$	β-*cut* of the fuzzy number $\tilde{P}$
z	estimate of the fuzzy number $\tilde{Z}$