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Continuous Optimization

A norm-relaxed method of feasible directions for finely discretized problems from semi-infinite programming ☆

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Abstract

In this paper, a class of finely discretized Semi-Infinite Programming (SIP) problems is discussed. Combining the idea of the norm-relaxed Method of Feasible Directions (MFD) and the technique of updating discretization index set, we present a new algorithm for solving the Discretized Semi-Infinite (DSI) problems from SIP. At each iteration, the iteration point is feasible for the discretized problem and an improved search direction is computed by solving only one direction finding subproblem, i.e., a quadratic program, and some appropriate constraints are chosen to reduce the computational cost. A high-order correction direction can be obtained by solving another quadratic programming subproblem with only equality constraints. Under weak conditions such as Mangasarian–Fromovitz Constraint Qualification (MFCQ), the proposed algorithm possesses weak global convergence. Moreover, the superlinear convergence is obtained under Linearly Independent Constraint Qualification (LICQ) and other assumptions. In the end, some elementary numerical experiments are reported.

Keywords: Semi-infinite programming; Norm-relaxed method of feasible direction; Global convergence; Superlinear convergence

1. Introduction

Optimization problems arising in engineer design often belong to the class of Semi-Infinite Programming (SIP) problems. For the sake of exposition, a simple example is given as follows

$$\text{SIP} \quad \min \quad f(x) \qquad \text{s.t.} \quad \Phi_{[0,1]}(x) \leqslant 0, \tag{1.1}$$

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with

$$\Phi_{[0,1]}(x) \triangleq \sup_{\omega \in [0,1]} \phi(x, \omega),$$

where $f : R^n \to R$ is continuously differentiable and $\phi : R^n \times [0,1] \to R$ is continuously differentiable with respect to x.

In recent decades, many efforts have been made in the researches of SIP. Especially since 1990s, the research of SIP has made a great development both in algorithm theory and performing algorithm. As to the early works of smooth SIP, Ref. [1] made an analysis in detail. Subsequently, basing on methods for smooth SIP, some authors proposed some algorithms for solving SIP, such as [2–6]. In fact, since entropy function is presented (see [7,8]), maximum entropy principle has been gradually applied to SIP, and the relative works are very abundant (see [9–11] etc.).

In fact, the methods of discretization are also effective for solving SIP. Many globally convergent algorithms are based on approximating $\Phi_{[0,1]}(x)$ by means of progressively finer discretization of the interval $[0,1]$ (see e.g. [12–17]). For example, given any $q \in \mathbb{N} \setminus \{0\}$, the interval $[0,1]$ can be discretized into the following finite set

$$\Omega = \left\{0, \frac{1}{q}, \frac{2}{q}, \ldots, \frac{q-1}{q}, 1\right\},$$

which is also called discretization index set. The constraint of SIP is approximated accordingly by a sequence of constraints $\phi(x, \omega) \leqslant 0,\ \forall \omega \in \Omega$. Thus, in the algorithms [12,17], solving SIP problem can be substituted by solving the following problem of the form

$$\begin{aligned} \text{DSI} \quad & \min \quad f(x) \\ & \text{s.t.} \quad \phi(x, \omega) \leqslant 0 \quad \forall \omega \in \Omega, \end{aligned} \tag{1.2}$$

which is said the Discretized Semi-Infinite (DSI) problem. Obviously, the algorithms [12–17] depend heavily on being able to efficiently solve problem DSI. Meanwhile, we note that q reflects the discretization level of DSI, and it can be progressively increased (see, e.g. [12,15–17]). The overall performance of these algorithms are crucially dependent upon the performance at each discretization level, especially when q becomes large. For example, in [12], the discretization is progressively refined and the corresponding problem is solved to a progressively better accuracy. Convergence of the algorithm to stationary points of SIP is proven. In addition, Ref. [16] proposes a simple modification scheme of the algorithm [15], and the iteration point X_k is updated every time the dicretization is refined, and the scheme [16] constructs an infinite sequence $\{X_k\}$ and every accumulation point of this sequence is a Kuhn–Tucker point for SIP.

Although problem DSI has the same form as a general inequality constrained optimization problem

$$\begin{aligned} \text{MC} \quad & \min \quad f(x) \\ & \text{s.t.} \quad \phi_j(x) \leqslant 0, \quad j = 1, 2, \ldots, m \ (m = q + 1), \end{aligned}$$

there are some difference between DSI and MC. The main difference is that problem DSI has "continuous" dependence of the constraint function on the index. An obvious consequence of this continuity is that a fine discretization produces "neighboring constraints" which are very similar, or numerically even identical. In other words, the constraints in DSI are "sequentially related" in the sense that the values taken by ϕ_i are typically close to those taken by ϕ_{i+1}. This can have advantages and disadvantages for numerical methods.

Of course, problem DSI involves only a finite number of smooth constraints and in theory can be solved by classical inequality constrained optimization technique. However, we note that when the discretization is fine, i.e., q is very large, the number of constraints becomes large, but only a small portion of the constraints are active at the solution. Suitably taking advantage of this situation may lead to substantial computational saving. Early efforts at employing such a scheme see [12,16], in which DSI is solved by means of first-order method of feasible directions. In [12], based on ideas of Zoutendijk, at iteration k, a search direction is computed by using only the gradients $\nabla_x \phi(x^k, \omega)$ at all points $\omega \in \Omega$ satisfying $\phi(x^k, \omega) > -\epsilon$, where $\epsilon > 0$ is