Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 186 (2008) 261–275
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Decision Support

Decision making with hybrid influence diagrams using mixtures of truncated exponentials

Barry R. Cobb [a,*], Prakash P. Shenoy [b]

[a] Department of Economics and Business, Virginia Military Institute, Scott Shipp Hall, Lexington, VA 24450, USA
[b] School of Business, University of Kansas, 1300 Sunnyside Avenue, Summerfield Hall, Lawrence, KS 66045-7585, USA

Received 11 March 2005; accepted 18 January 2007
Available online 20 February 2007

Abstract

Mixtures of truncated exponentials (MTE) potentials are an alternative to discretization for representing continuous chance variables in influence diagrams. Also, MTE potentials can be used to approximate utility functions. This paper introduces MTE influence diagrams, which can represent decision problems without restrictions on the relationships between continuous and discrete chance variables, without limitations on the distributions of continuous chance variables, and without limitations on the nature of the utility functions. In MTE influence diagrams, all probability distributions and the joint utility function (or its multiplicative factors) are represented by MTE potentials and decision nodes are assumed to have discrete state spaces. MTE influence diagrams are solved by variable elimination using a fusion algorithm.

Keywords: Decision analysis; Influence diagrams; MTE potentials; Applied probability

1. Introduction

An influence diagram is a compact graphical representation for a decision problem under uncertainty. Initially, influence diagrams were proposed as a front-end for decision trees (Howard and Matheson, 1984). Subsequently, Olmsted (1983) and Shachter (1986) developed methods for evaluating an influence diagram directly without converting it to a decision tree. These methods assume that all uncertain variables in the model are represented by discrete probability mass functions (PMF's). Several improvements to solution procedures for solving discrete influence diagrams have been proposed (see, e.g., Shenoy, 1992; Shachter and Ndilikilikesha, 1993; Jensen et al., 1994; Madsen and Jensen, 1999; Lauritzen and Nilsson, 2001; Madsen and Nilsson, 2001).

Shachter and Kenley (1989) introduced Gaussian influence diagrams, which contain continuous variables with Gaussian distributions and a quadratic value function. In this framework, chance nodes have conditional

* Corresponding author. Tel.: +1 540 464 7452.
E-mail addresses: cobbbr@vmi.edu (B.R. Cobb), pshenoy@ku.edu (P.P. Shenoy).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2007.01.036

linear Gaussian distributions, meaning each chance variable has a Gaussian distribution whose mean is a linear function of the variable's parents and whose variance is a constant. This framework does not allow discrete chance nodes; however, it does allow chance variables whose distributions are conditionally deterministic linear functions of their parents.

Poland and Shachter (1993) introduce mixture of Gaussians influence diagrams, which allow both discrete and continuous nodes where continuous variables are modeled as mixtures of Gaussians. In this framework, instantiating all discrete nodes reduces the model to a Gaussian influence diagram. The influence diagram must satisfy the condition that discrete chance nodes cannot have continuous parents. In this model, a quadratic value function is specified along with a utility function which represents risk-neutral behavior or a constant risk aversion. Poland (1994) proposes a procedure for solving such influence diagrams which uses discrete and Gaussian operations and reduces continuous chance variables before discrete chance variables. Madsen and Jensen (2005) describe a new procedure for exact evaluation of similar influence diagrams that contain an additively decomposing quadratic utility function.

Monte Carlo methods have also been proposed for solving decision problems with continuous and discrete variables. Bielza et al. (1999) uses Markov chain Monte Carlo methods to solve *single stage* problems with continuous decision and chance nodes. Charnes and Shenoy (2004) solve *multiple stage* decision problems using a multi-stage Monte Carlo sampling technique that takes advantage of local computation to limit the number of variables sampled at one time.

Mixtures of truncated exponentials (MTE) potentials are suggested by Moral et al. (2001) and Rumí (2003) as an alternative to discretization for solving Bayesian networks with a mixture of discrete and continuous chance variables. In this paper, we propose MTE influence diagrams, which are influence diagrams in which probability distributions and utility functions are represented by MTE potentials. We solve MTE influence diagrams using the fusion algorithm proposed by Shenoy (1993) for the case where the joint utility function decomposes multiplicatively.

The remainder of this paper is organized as follows. Section 2 introduces notation and definitions used throughout the paper. Section 3 defines MTE potentials. Section 4 reviews the operations required for solving an MTE influence diagram. Section 5 presents details of a method for approximating joint utility functions with MTE utility potentials. Section 6 contains an adaptation of Raiffa's (1968) Oil Wildcatter problem, which is represented and solved using an MTE influence diagram. Finally, Section 7 summarizes and states some directions for future research.

2. Notation and definitions

This section contains notation and definitions used throughout the paper.

2.1. Notation

Variables will be denoted by capital letters, e.g., A, B, C. Sets of variables will be denoted by boldface capital letters, $\mathbf{Y}$ if all are discrete chance variables, $\mathbf{Z}$ if all are continuous chance variables, $\mathbf{D}$ if all are decision variables, or $\mathbf{X}$ if the components are a mixture of discrete chance, continuous chance, and decision variables. In this paper, all decision variables are assumed to be discrete. If $\mathbf{X}$ is a set of variables, $\mathbf{x}$ is a configuration of specific states of those variables. The discrete, continuous, or mixed state space of $\mathbf{X}$ is denoted by $\Omega_{\mathbf{X}}$.

MTE probability potentials and discrete probability potentials are denoted by lower-case greek letters, e.g., α, β, γ. Discrete probabilities for a specific element of the state space are denoted as an argument to a discrete potential, e.g. $\delta(0) = P(D = 0)$. MTE utility potentials are denoted by u_i, where the subscript i indexes both the initial MTE utility potential(s) specified in the influence diagram and subsequent MTE utility potentials created during the solution procedure.

In graphical representations, decision variables are represented by rectangular nodes, discrete chance variables are represented by single-border ovals, continuous chance variables are represented by double-border ovals, and utility functions are represented by diamonds.