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An improved algorithm for the computation of the optimal repair/replacement policy under general repairs

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Abstract

We consider a system which deteriorates with age and may experience a failure at any time instant. On failure, the system may be replaced or repaired. The repair can partially reset the failure intensity of the unit. Under a suitable cost structure it has been proved in the literature that the average-cost optimal policy is of control-limit type, i.e. it conducts a replacement if and only if, on the nth failure, the real age of the system is greater than or equal to a critical value. We develop an efficient special-purpose policy iteration algorithm that generates a sequence of improving control-limit policies. The value determination step of the algorithm is based on the embedding technique. There is strong numerical evidence that the algorithm converges to the optimal policy.

Keywords: Dynamic programming; Repair; Replacement; Average cost

1. Introduction

A great number of stochastic models have been introduced to describe the behavior of a repairable system that is subject to failure. In most of these models (see e.g. [1,7,11,9,10]) it was assumed that there are only two types of repair, the perfect repair and the minimal repair. The former results in a functioning system, which is as good as new, while the latter restores the system to its functioning condition just prior to failure.

Kijima et al. [3] studied the general repair model, in which the repair brings the state of the system to a certain better state. The minimal repair and the perfect repair are two special cases. They assumed that the repair and replacement costs are constant and considered a periodical replacement problem where the system is replaced at only scheduled times kT, $k = 0, 1, \ldots$ and is repaired whenever it fails. The long-run average cost per unit time was derived and an approximation procedure, which can be used to find the optimal replacement period, was proposed.

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Makis and Jardine [6] generalized the above repair/replacement process by formulating a suitable semi-Markov decision model. The system failure instants are the decision epochs. A two dimensional infinite state space was utilized, where the state of the process consists of the number of failures and the real age of the system. The first state variable is discrete while the second one is continuous. The replacement cost was assumed to be fixed and the repair cost was assumed to depend on the number of failures and the age of the system. Under some conditions on the costs and the failure rate of the system, Makis and Jardine showed that the policy that minimizes the long-run expected average cost per unit time is of control-limit type, i.e. it replaces the system at the nth failure if and only if its age is greater than or equal to some critical value that depends on n.

Love et al. [5] approximated the semi-Markov decision model of Makis and Jardine with a finite-state discrete semi-Markov decision model, by truncating the state space and by discretizing the second state variable. They determined the one-step transition probabilities and transition times for the reformulated model and developed an algorithm that generates a sequence of strictly improving control-limit policies. In the last paragraph of Section 3 of their paper, they mention that the algorithm determines the optimal control-limit policy. As we will explain in Section 3 of the present article, there is no proof of this assertion, though there is strong numerical evidence that it is true.

The main purpose of the present paper is to improve the algorithm of Love et al. by applying Tijms's [13, p. 234] embedding technique. In our problem it is possible to find explicit expressions for all quantities that are needed for the application of the embedding technique. This technique reduces considerably the calculations in the value determination step of the algorithm, which have been increased considerably because of the discretization of the second state variable.

The rest of the paper is organized as follows. The description of the problem and the relevant finite-state semi-Markov decision model are given in the next section. In Section 3 we develop the special-purpose policy iteration algorithm by applying Tijms's embedding technique. Some numerical results are presented in Section 4.

2. Model formulation

Consider a system that deteriorates over time and is subject to failures. The state of the system is represented by the pair of variables (n, t_n), where n denotes the nth failure and t_n the real age of the system at that instant. The first variable (n) is discrete and the second one (t_n) is continuous. The state space is the following continuous set:

$$I = \{(n, t_n) | n = 1, 2, \ldots, t_n \geqslant 0\}.$$

A maximum number N of failures before replacement and a number B, as the upper bound on the real age of the system, are assumed. Following Love et al. [5] we discretize the second state variable as follows. The real age axis is divided into a set of equally spaced age slices and we regard that the nth failure occurs at the age slice i_n. A scaling parameter ξ is also defined as the number of time slices in a real time unit to relate the time slice i_n to the real time t_n. Thus, if the nth failure occurs at the age slice i_n, we assume that the real age of the system at that failure instant is between i_n/ξ and $(i_n + 1)/\xi$.

Henceforth we suppress the subscript n on the age slices i_n and simply refer to the age slice as i. The state space I is approximated by the following finite and discrete set:

$$S = \{(n, i) | 1 \leqslant n \leqslant N, 0 \leqslant i \leqslant M\},$$

where $M = B\xi$. Note that we can increase the accuracy of this approximation by increasing the values of N, B, ξ.

The decision epochs are the system failure instants. At each decision epoch two actions $a \in \{0, 1\}$ may be taken. The system either can be repaired ($a = 1$) or can be replaced by a new identical one ($a = 0$). It is assumed that both maintenance activities are executed in negligible time.

In states (N, i),$0 \leqslant i \leqslant M$, and (n, M),$1 \leqslant n \leqslant N$, the action of replacement is mandatory and brings the system to the states $(1, j)$, where $0 \leqslant j \leqslant M$. If at a decision epoch, the system is in states (n, i), $1 \leqslant n \leqslant N - 1$, $0 \leqslant i \leqslant M - 1$, and the action of repair is selected, the system makes a transition to the states $(n + 1, j)$, where