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Optimal ordering policies for periodic-review systems with a refined intra-cycle time scale

Chi Chiang *

Department of Management Science, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan, ROC

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Abstract

Chiang [C. Chiang, Optimal ordering policies for periodic-review systems with replenishment cycles, European Journal of Operational Research 170 (2006) 44–56] recently proposed a dynamic programming model for periodic-review systems in which a replenishment cycle consists of a number of small periods (each of identical but arbitrary length) and holding and shortage costs are charged based on the ending inventory of small periods. The current paper presents an alternative (and concise) dynamic programming model. Moreover, we allow the possibility of a positive fixed cost of ordering. The optimal policy is of the familiar (s, S) type because of the convexity of the one-cycle cost function. As in the periodic-review inventory literature, we extend this result to the lost-sales periodic problem with zero lead-time. Computation shows that the long-run average cost is rather insensitive to the choice of the period length. In addition, we show how the proposed model is modified to handle the backorder problem where shortage is charged on a per-unit basis irrespective of its duration. Finally, we also investigate the lost-sales problem with positive lead-time, and provide some computational results.

Keywords: Inventory; Lost sales; Periodic-review systems; Dynamic programming

1. Introduction

Periodic-review inventory systems are commonly found in practice, especially if many different items are purchased from the same supplier and the coordination of ordering and transportation is important. In a recent survey [7, p. 69], material managers indicate the effectiveness of periodic-review systems for reducing inventory levels in a supply chain.

Although most studies on periodic-review inventory models have (implicitly) assumed that the review periods are as small as one day (see, e.g., [5] and references therein), periodic-review systems in practice often have the review periods (i.e., *replenishment cycles* or simply *cycles*) that are a few days or weeks long and regular orders are placed at a review epoch (see, e.g., [2,3] for periodic systems where an emergency order can be placed at a review epoch or virtually at any time between two review epochs). For such periodic-review

* Tel.: +886 35 712121x5201.
E-mail address: cchiang@mail.nctu.edu.tw

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systems, it is appropriate to compute holding and shortage costs based on respectively, the average inventory of a replenishment cycle and the duration of shortage (assuming that demand not immediately filled is backlogged). Due to the difficulties involved in exact analysis, the approximate treatment of such systems is often used in textbooks (e.g., [4, Sec. 5-2] and [6, Sec. 7.9.4]) to obtain easy-to-implement solutions. However, as Chiang [1] recently pointed out, there are many shortcomings with the approximate treatment. Chiang thus proposed a dynamic programming model in which a cycle consists of a number of small periods and holding and shortage costs will be computed based on the ending inventory of small periods (rather than only on the ending inventory of cycles). As periods can be chosen to be any time units (see [1] for a detailed discussion), the period length is tailored to the needs of an application.

In this paper, we present an alternative (and concise) dynamic programming model. Moreover, we allow the possibility of a positive fixed ordering cost that is not considered in [1]. The optimal policy is of the familiar (s, S) type, i.e., if inventory drops to or below s at a review epoch, an order is placed to raise the inventory to a predetermined level S. As in the periodic-review inventory literature (see, e.g., [9]), we extend this result to the lost-sales periodic problem with zero lead-time. Computation shows that the long-run average cost is rather insensitive to the choice of the period length. This indicates that a rough estimate of the period length is acceptable.

In addition, we show how the proposed model is modified to handle the backorder problem where shortage is charged on a per-unit basis irrespective of its duration (as in [4, Sec. 5-2]), which is also not considered in [1]. Finally, we investigate the lost-sales problem with positive lead-time. Although we are unable to derive any properties (as in [1]) that can be used in the dynamic programming computation, we provide some interesting computational results. We advocate that firms use the proposed models for obtaining optimal ordering policies.

2. The backorder model

Suppose that a replenishment cycle, whose length is exogenously determined (as in [1–3]), consists of m periods, each of identical but arbitrary length. Let ξ be a generic demand variable and in particular, let ζ denote the demand of a cycle. Also, let $\varphi^k(\cdot)$ be the probability density function of k-period's demand (the superscript may be omitted for brevity if $k = 1$). Assume first that all demand not immediately satisfied is backlogged. Demand is assumed to be non-negative and independently distributed in disjoint time intervals. In addition, the following notation is used.

λ	mean arrival rate
τ	the (deterministic) supply lead-time, which is an integral multiple of a period
c	the unit procurement cost
K	the fixed cost of ordering
hc	the inventory cost per unit held per cycle
h	the inventory cost per unit held per period (i.e., $h = \mathrm{hc}/m$)
pc	the shortage cost per unit per cycle
p	the shortage cost per unit per period (i.e., $p = \mathrm{pc}/m$)
π	the shortage cost per unit
L	the holding and shortage costs of a replenishment cycle
α	the one-period discount factor (i.e., discounting the cost incurred in one period from now to the present time), $0 < \alpha \leqslant 1$
$\delta(\cdot)$	1 if the argument is positive and 0 otherwise
X	the starting inventory position (i.e., inventory on hand minus backorder plus inventory on order) at a review epoch
$V_n(X)$	the expected discounted cost of ordering, procurement, holding and shortage with n cycles remaining until the end of the planning horizon, given X at a review epoch and an optimal policy is used

For simplicity of formulation, we exclude from $V_n(X)$ the holding and shortage costs during the next τ periods, because these costs are not affected by the decision made at a review epoch. $V_n(X)$ satisfies the functional equation