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Discrete time market with serial correlations and optimal myopic strategies

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Abstract

The paper studies discrete time market models with serial correlations. We found a market structure that ensures that the optimal strategy is myopic for the case of both power or log utility function. In addition, discrete time approximation of optimal continuous time strategies for diffusion market is analyzed. It is found that the performance of optimal myopic diffusion strategies cannot be approximated by optimal strategies with discrete time transactions that are optimal for the related discrete time market model.

Keywords: Control; Economics; Stochastic processes; Optimal portfolio

1. Introduction

The paper investigates discrete time stochastic market models. We consider an optimal investment problem that includes as a special case a problem where $\mathbf{E}U(X_T)$ is to be maximized, where X_T represents the total wealth at final time T and where $U(\cdot)$ is a utility function. We consider two types of utility function: $U(x) = \ln x$ and $U(x) = \delta^{-1}x^{\delta}$, where $\delta < 1$ and $\delta \neq 0$. For continuous time market models, these utilities have a special significance, in particular, because the optimal strategies for them can be *myopic*. In that case they do not require future distributions of parameters and do not depend on terminal time. In fact, the optimal strategies for power utilities for continuous time are myopic under some additional assumptions when the risk free rate, the appreciation rate, and the volatility matrix are random processes that are supposed to be currently observable (may be, with unknown prior distributions and evolution law). Besides, these parameters must be independent of the driving Brownian motion (i.e., it is the case of ''totally unhedgeable'' coefficients, according to Karatzas and Shreve (1998), Chapter 6). The solution that leads to myopic strategies goes back to Merton (1969); the case of random coefficients was discussed in Karatzas and Shreve (1998) and Dokuchaev and Haussmann (2001).

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The real stock prices are presented as time series, so the discrete time (multi-period) models are more natural than continuous time models. On the other hand, continuous-time models give a good description of distributions and often allows explicit solutions of optimal investment problems.

For the real market, a formula for an optimal strategy derived for a continuous-time model can often be effectively used after the natural discretization. However, this strategy will not be optimal for time series observed in the real market. Therefore, it is important to extend the class of discrete time models that allow myopic and explicit optimal portfolio strategies. The problem of discrete-time portfolio selection has been studied in the literature, such as in Smith (1967), Chen et al. (1971), Leland (1968), Mossin (1968), Merton (1969), Samuelson (1969), Fama (1970), Hakansson (1971a,b), Elton and Gruber (1974, 1975), Winkler and Barry (1975), Francis (1976), Dumas and Liucinao (1991), Östermark (1991), Grauer and Hakansson (1993), Pliska (1997), and Li and Ng (2000). If a discrete time market model is complete, then the martingale method can be used (see, e.g., Pliska, 1997). Unfortunately, a discrete time market model can be complete only under very restrictive assumptions. For incomplete discrete time markets, the main tool is dynamic programming that requires solution of Bellman equation starting at terminal time. For the general case, this procedure requires to calculate the conditional densities at any step (see, e.g., Pliska, 1997 or Gikhman and Skorohod, 1979). This is why the optimal investment problems for discrete time can be more difficult than for continuous time setting that often allows explicit solutions.

There are several special cases when investment problem allows explicit solution for discrete time, and, for some cases, optimal strategies are myopic (see Leland, 1968; Mossin, 1968; Hakansson, 1971a,b). However, the optimal strategy is not myopic and it cannot be presented explicitly for power utilities in general case. Hakansson (1971a,b) showed that the optimal strategy is not myopic for $U(x) = \sqrt{x}$ if returns have serial correlation and evolve as a Markov process.

In the present paper, we study the optimal investment problem for a incomplete discrete time market under some general assumptions. We found a wide class of models with serial correlation such that the optimal strategies are myopic for both power and log utilities. In fact, the basic restrictions for this class of models are similar to the ones that ensure optimality of myopic strategy in continuous setting. We present an algorithm for calculation of optimal strategies. These strategies are analogs of Merton's optimal strategies for diffusion market model. Note that these strategies are different from the strategies constructed via the natural discretization of the Merton's strategies.

In addition, we found the following interesting consequence: the difference between the optimal expected utilities for discrete time and continuous time models does not disappear if the number of periods (or frequency of adjustments) grows. In particular, we found that the optimal expected utility calculated for continuous time market cannot be approximated by piecewise constant strategies with possible jumps at given times $\{t_k\}_{k=1}^{T}$, even if $T \to +\infty$ and $t_k - t_{k+1} \to 0$ (see Corollary 5.1 below).

Our model includes the case when the risk-free rate may have correlation with the risky asset. For simplicity, we considered single stock market only, but the generalization for the multi-stock case is straightforward.

2. The market model

We consider a model of a market consisting of the risk-free bond or bank account with price B_k and the risky stock with price S_k, $k = 0, 1, 2, \ldots, T$, where $T \geqslant 1$ is a given integer. The initial prices $S_0 > 0$ and $B_0 > 0$ are given non-random variables.

We assume that

$$\begin{aligned} S_k &= \rho_k S_{k-1}(1 + \xi_k), \\ B_k &= \rho_k B_{k-1}, \quad k = 1, 2, \ldots, T. \end{aligned} \tag{1}$$

Here ξ_k and ρ_k are random variables. We assume that $\xi_k > -1$ and $\rho_k \geqslant 1$ for all k.

We are given a standard probability space $(\Omega, \mathscr{F}, \mathbf{P})$, where Ω is the set of all elementary events, $\mathscr{F}$ is a complete σ-algebra of events, and $\mathbf{P}$ is the probability measure.

Let us describe our main assumptions about the distributions of ξ_k and ρ_k.

Let $\mathscr{Z}$ be a metric space.

We assume that the following condition is satisfied.