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Decision Support

Fuzzy trees in decision support systems

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Abstract

This paper is based on the following assumption: that there exists a fuzzy tree structure and a distance between fuzzy trees which provides the basis for fuzzy decision-making. The paper provides the following: (1) a new definition of the fuzzy relational tree structure, (2) the development of a new comparative method for fuzzy trees and its experimental testing and evaluation, (3) a new descriptive method of military structures in a fuzzy tree format and the development of a fuzzy decision support system.

Keywords: Fuzzy sets; Graph theory; Decision support systems

1. Introduction

1.1. Decision-making in complex and ill-defined systems

Complex systems and ill-defined information are one of the most influential factors in the decision-making process. When making a decision, one has to face the uncertainty of future events and the uncertainties which accompany the transmission, transfer and reception of information.

Classical approaches to system modelling can hardly cope with complex systems and systems involving uncertainty for it is extremely difficult to find a global function or analytical structure for a non-linear system. Fuzzy set theory provides a mathematical frame for system modelling when knowledge and information about a system are incomplete and experience-based rather than systematic. Such problems are often encountered when applied to control, pattern recognition or MCDM (Andriole, 1990; Carlsson, 1994).

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Therefore, it is obvious that in real life many processes are closer to fuzzy processes than mathematically precisely defined procedures. One often gets the impression that, in real life situations, problems are most efficiently solved using the human factor because machines (computers) have proved too complicated and impractical. As a consequence, there is an increasing need to improve "machine" capacities so that we can solve fuzzy decision-making problems. These can be defined as making decisions in a fuzzy environment.

Many methods that attempt to answer the problem of decision-making have been developed. Most of them have so far been based mainly on probability calculus, statistics, mathematical linear and non-linear programming and similar classical mathematical tools. However, the increasing complexity of systems on the one hand, and a lack of information on the other hand, demand new approaches and methods for the decision-making process. Modern approaches to decision-making are based on structural descriptions of objects and the use of expert systems. One of them, fuzzy logic, has the advantage of a better overview of ill-defined systems compared to classical data-probability approaches.

In accordance with such principles, we suggest a new method of solving fuzzy decision-making problems. Data presentation in the shape of trees enables us to save structural information in the form of structures with hierarchical relations between basic elements. Such models have proved very efficient as support for decisions. Due to their mathematical characteristics they can be easily manipulated using efficient algorithms.

In this paper we present a decision support system that is based on the comparison of fuzzy trees. As an example of a decision-making process we introduce a fuzzy system that supports decision-making for the commander of a combat unit.

1.2. Fuzzy relational structures

Graph theory plays an important role in the modelling of complex structures. There are three definitions of graph fuzzification: fuzzification of nodes, fuzzification of relations and simultaneous fuzzification of nodes and relations. Fuzzy trees, which are a special case of fuzzy graphs, are helpful for representing soft or ill-defined structures, especially in decision analysis and in decision support systems.

When the knowledge about the system is incomplete, or when the system is complex and data is lacking, or when we deal with information that is either not correct or even intentionally false, the principles of fuzzy set theory can be applied. The theory of fuzzy graphs and trees has important links to the theory of fuzzy classification and decision analysis. The distance between two trees is defined in terms of the least number of substitutions, insertions, and deletions of nodes required to obtain one from the other. In this paper we propose the language transformation of fuzzy trees, namely, substitution, deletion and insertion of fuzzy nodes.

With the increasing popularity of using hierarchical data structure and tree construction schemes, we have to face the problem of comparing two tree-like data structures. The problem may arise in several ways: from comparing the tree structure of an input with that of the templates for classification and decision support, or from sorting the data according to the similarity between tree structures for fast retrieval. These situations initiated the study of distance measures for trees.

2. Fundamentals of fuzzy set theory

2.1. Some conceptual foundations

In the last few years, fuzzy set theory has become of practical value in fields which were not long ago considered to be the domain of classical methods. As a consequence, fuzzy sets are being increasingly applied to decision-making, management, planning, anticipation and optimisation. Information itself, which