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Continuous Optimization

Optimal search and ambush for a hider who can escape the search region

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ABSTRACT

Search games for a mobile or immobile hider traditionally have the hider permanently confined to a compact 'search region' making eventual capture inevitable. Hence the payoff can be taken as time until capture. However in many real life search problems it is possible for the hider to escape an area in which he was known to be located (e.g. Bin Laden from Tora Bora) or for a prey animal to escape a predator's hunting territory. We model and solve such continuous time problems with escape where we take the probability of capture to be the searcher's payoff.

We assume the searcher, while cruise searching, can cover the search region at unit rate of area, for a given time horizon T known to the hider. The hider can stay still or choose any time to flee the region. To counter this, the searcher can also adopt an ambush mode which will capture a fleeing hider. The searcher wins the game if he either finds the hider while cruise searching or ambushes him while he is attempting to flee; the hider wins if he flees successfully (while the searcher is cruising) or has not been found by time T. The optimal searcher strategy involves decreasing the ambush probability over time, to a limit of zero. This surprising behaviour is opposite to that found recently by Alpern et al. (2011, 2013) in a predator-prey game with similar dynamics but without the possibility of the hider escaping. Our work also complements that of Zoroa et al. (2015) on searching for multiple prey and Gal and Casas (2014) for a combined model of search and pursuit.

1. Introduction

Since their introduction in the classical text of Isaacs (1965), search games have proved a useful method of modelling optimal search for a mobile or immobile antagonistic hider who is confined to a bounded search region $\mathcal{R}$. Even for mobile hiders, it was shown by Gal (1979); 1980) for multidimensional regions and Alpern and Asic (1985) for finite length networks, that eventual capture is almost surely accomplished and moreover capture time has finite expectation. Hence such games have been traditionally solved by taking capture time (search time) as the payoff of a zero sum game.

However in many real life problems, the hider is able to at least attempt to leave the region in which he is initially known to be confined. For example, Osama Bin Laden successfully escaped from the Tora Bora caves, where he was at one time known to be hiding (see Weaver 2005). In the predator-prey context, it is possible for the hider (prey animal) to escape the hunting territory of the predator who is searching for it. In such a context of potential escape, a more reasonable searching aim is to maximize the probability of eventually finding the hider, placing less emphasis on the search time but more on the search outcome. Here we initiate the study such problems where the searcher has a limited time horizon T in which to capture the hider and the eventual outcome is uncertain.

Thus we are led to a continuous time game, where the hider can stay still or choose any time m, $0 \leq m \leq T$, in which to attempt a flight from the search region. To counter this possibility, the searcher has an additional 'ambush' mode, in which he can counter an attempt at flight. In the predator-prey context, the ambush mode might involve sitting still and surveying the search region for a move of the prey, or an eagle circling above the region to spot the prey if it goes out of the vegetation cover in an attempt to leave the region. For law enforcement, an ambush mode might involve setting up road blocks to counter an attempted location change of an escaped prisoner (who has escaped prison but

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not the surrounding search region). In a search game context on a network, ambush strategies might consist of the searcher waiting at a node (e.g. the central node of a star network) to catch a mobile hider. Such search strategies were initially excluded by Gal (1979) in order to obtain certain results, but later incorporated into the theory by Alpern and Asic (1986), who showed that in the figure eight network they had to be present in optimal search strategies. Such strategies were shown to be important in a predator-prey context for dual mode predators (who can alternate between a 'sit and wait' mode and a cruising mode) by Alpern, Fokkink, Gal, and Timmer (2013); Alpern, Fokkink, Timmer, and Casas (2011). Thus our game has four possible outcomes. The two outcomes where the hider wins are (i) where the hider never attempts to flee and is not found by time T, and (ii) where he successfully flees because the searcher is cruise-searching at the flight time m. The two outcomes where the searcher wins are when he (iii) finds the hider while cruise-searching, before any hider attempt to flee, and when he (iv) ambushes the hider when the latter attempts to flee, by adopting the ambush mode at time m.

To give the reader some additional intuition about our model, we mention an interpretation of the a discrete time version given in Section 5 as a smuggling or search-inspection game: A smuggler (hider) brings his material randomly to one of n identical warehouses on the left side of a river. The police are known to be in the area for T days. On any of these days, or on day $T+1$, the smuggler can attempt to cross the river to the safe right side. Similarly, on each day the police can either search one of the warehouses or alternatively patrol the river, but not both. The police apprehend the smuggler if on one of the T days they either find the material in the warehouse they search, or are patrolling and the smuggler attempts to cross the river. If at the end of the T days the smuggler has not attempted a crossing and he has not been found, he can cross without fear on the next day, so he wins. The smuggler also wins if he crosses on a day when the police are searching a warehouse. We need to add the 'noisy searcher' assumption that the smuggler, safe in his warehouse, can hear whether or not a speedboat is patrolling the river, so each morning he knows how many warehouses have been searched thus far. The 'silent' version, with a canoe instead of a speedboat, is a harder problem, still open. Some progress in this context was made by Arcullus (2013).

While the notion that the sought after hider has the possibility of fleeing the search region is new, the notion of an ambush strategy for the searcher has been studied in the literature. Originally introduced for search games in Alpern and Asic (1986), various forms of ambush problems have been explored in Baston and Bostock (1987), Hohzaki and Iida (2001), Baston and Kikuta (2004), and Zoroa, Fernández-Sáez, and Zoroa (2011); Zoroa, Zoroa, and Fernández-Sáez (1999). In the predator-prey context, the ambush mode of certain predators is often called the 'sit-and-wait' strategy. The alternation of predators between cruise searching and ambush search has been studied by Alpern et al. (2011); Arcullus (2013), Zoroa, Fernández-Sáez, and Zoroa (2015) and Arcullus (2013). The biological context has received additional attention in a wider context by Broom (2013), Pitchford (2013) and Gal and Casas (2014), where hide-search is combined with pursuit-evasion in a novel way. Djemai, Meyhöfer, and Casas (2000) gives related empirical work on search problems between a host and a parasitoid. Other recent work on search games includes the more abstract work of Oléron Evans and Bishop (2013) and the operational research related approach of Zoroa, Fernández-Sáez, and Zoroa (2012); Zoroa, Zoroa, and Fernández-Sáez (2009). The two monographs on search games are Garnaev (2000) and Alpern and Gal (2003). Related work on patrolling is given in Lin, Atkinson, Chung, and Glazebrook (2013) and Zoroa et al. (2009). Connections with inspection and smuggling games are mentioned in Section 5, on a discrete version of our game.

2. The dynamic model

We begin by describing a continuous time dynamic model of search and ambush that is similar to that introduced in Alpern et al. (2013, 2011). A hider (prey) is hidden at a random point of a unit size (area) search region $\mathcal{R}$. The searcher (predator) can search $\mathcal{R}$ at a unit time rate for time T, so that all of $\mathcal{R}$ could be searched by time 1 (if $T > 1$). However the hider can counter exhaustive search by *fleeing* (leaving the region $\mathcal{R}$) at a time $t = m$ of his choice. In this model, successful flight (during a period when the searcher is cruising) wins the game for the hider. To counter this possibility, the searcher can adopt an *ambush* mode which will catch a fleeing hider, winning for the searcher. The game ends, as a win for the searcher, if the searcher either finds the hider before he flees or if he successfully ambushes the hider while he is fleeing. Thus far the dynamics of our model are the same as in Alpern et al. (2011). However in that model a successful (unambushed) flight by the hider simply gives him a new randomized location in the region $\mathcal{R}$, leading to a repetition of the stage game. In the current model a successful flight takes the hider outside of $\mathcal{R}$ to a safe location, and is thus considered a win for him. In the previous model eventual capture of the hider was ensured, only its time was in doubt, and the searcher's aim was to minimize the search time. In the current model, eventual capture is not assured, and the searcher's aim is to maximize the probability of capture. Thus the current model is a win-lose game where capture (by either by finding the prey while cruising or ambushing it while it is fleeing) is a win for the searcher (payoff 1) and successful flight (during a cruising period of the searcher) a win for the hider (payoff 0). More generally, the payoff (to the maximizing searcher) is the probability of a capture, and the value V of the game is the optimal probability of capture, given best play on both sides. If the searcher has an unlimited time horizon in which to capture the prey, he can win with as high a probability as he likes. For example if he ambushes 99% of the time, randomly placed within each unit interval, he will still eventually search the whole region $\mathcal{R}$. Consequently the hider cannot afford remain still forever. But whenever he flees, he will face an ambushing searcher and be captured, with 99% probability. So we make the reasonable assumption that the searcher has a limited time T in which to make the capture, perhaps this is the length of the daylight period. We analyse this game $\Gamma(T)$.

3. The limited time game $\Gamma(T)$

We formally describe the game $\Gamma(T)$ where the searcher tries to capture the hider within a fixed time horizon T, either by finding him or ambushing him. As described in Alpern et al. (2013, 2011) the alternation between searching and ambushing can be modelled very simply by a search strategy $s(t)$, $0 \le t \le T$ which measures the amount (or equivalently, fraction) of $\mathcal{R}$ that the searcher has covered by time t, given that he covers area at unit rate while cruising and at zero rate while ambushing. We have the restriction $s(t_2) - s(t_1) \le t_2 - t_1$ as well as the initial condition $s(0) = 0$. Suppose that in a small time interval $J = [t, t + \Delta t]$ the searcher adopts a search mode one third of the time and an ambush mode two thirds of the time, perhaps randomly in many subintervals. Then the total area of $\mathcal{R}$ searched during the time interval J will be $\Delta t/3$, so we may describe s on this interval by $s'(t) = 1/3,$ and $s(t + \Delta t) - s(t) = \Delta t/3$. A smooth function s is considered as the uniform limit of alternations between cruising and ambushing, as described more specifically in the earlier papers. In general, we interpret $s'(t)$ as the probability that the searcher is searching at time