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Decision Support

Electricity retail contracting under risk-aversion

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ABSTRACT

Risk has always been a dominant part of financial decision making in any industry. Recently models, tools and computational techniques have been developed so that we can effectively incorporate risk in optimal decision policies. The focus of this paper is on electricity markets, where much of the inherent risk falls on the retail sector. We introduce a three-stage model of an electricity market where firms can choose to enter the retail market, then enter into retail contracts, and finally purchase electricity in a wholesale market to satisfy their contracts. We explicitly assume that firms are risk-averse in this model. We demonstrate how the behaviour of firms change with risk-aversion, and use the example of an asset-swap policy over a transmission network to demonstrate the importance of modeling risk-aversion in determining policy outcomes.

1. Introduction

Much of the classic literature on electricity markets is based on published models that assume firms act in a risk-neutral manner. Results such as the importance of long-term contracts in lowering spot market prices (Allaz & Vila, 1993; Carlton, 1979) and the efficiency of nodal pricing markets (Caramanis, 1982) were proved using this assumption. However risk is a dominant part of any financial decision making and if firms are risk-averse then many of the policy predictions based on risk-neutrality become suspect. For example, Neuhoff and De Vries (2004) show that generation investment is not necessarily efficient in equilibrium if consumers and investors are risk-averse.

Much of the risk inherent in electricity markets falls on the retail sector. Retailers typically form fixed price, variable volume (FPVV) contracts with their customers (residential and commercial consumers). This places the risk of supplying the electricity on the retailer, exposing them to a variety of sources of uncertainty through the wholesale market. Electricity wholesale markets (also known as spot markets) operate through optimal dispatch of generators to meet the demand at minimum cost while complying with transmission constraints (Schweppe, Caramanis, Tabors, & Bohn, 1988). Locational marginal prices of electricity are extracted from such optimization problems as the total cost to the system of meeting one more unit of demand at a location. As a result, the retail firms face price fluctuations from transmission or plant outages, fuel cost shocks, or direct risk from demand shocks.

In many markets, retailers vertically integrate or engage in contracts with generators to mitigate their spot price risk.[1] Even prior to the prevalence of electricity markets, Hobbs (1995) investigated electric utility resource planning in an optimization setting, discussing competition, price responsive demand and uncertainty. More recently, Anderson and Hu (2008) consider the strategic incentives for retailers to offer forward contracts to generators in a risk-neutral setting, showing that retailers would be potentially willing to pay a premium for forward contracts, in order to reduce prices in the wholesale market. Oliveira, Ruiz, and Conejo (2013) also consider contracting, but from the perspective of market design. They consider the coordination of a supply chain with multiple generators and retailers under different market structures. The few papers that explicitly examine risk-averse firms usually focus on long-term contract formation by generators and avoid modeling the retail market, although Aïd, Chemla, Porchet, and Touzi (2011) model the retail sector, incorporating risk using a mean-variance approach. However, within these papers there is little discussion of transmission risk, which is arguably critical to pricing in nodal markets, and limited discussion of any kind of competition or entry into retail markets.

In this paper, we introduce a model of an electricity market where risk-averse gentailers and retailers act to maximize profit in a three-stage game. We explicitly model the formation of contract prices in the retail sector, allowing for strategic behaviour by firms. Furthermore, firms have the option of choosing whether or not to enter the retail market in the first place (including those firms who own generation), making vertical integration (or lack thereof) an endogenous

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[1] This can also provide a hedge for generators where fuel costs are uncertain.

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outcome of the model. In order to model risk the firms maximize profit while minimizing conditional value-at-risk (CVaR).

The existing literature on risk-aversion in electricity markets can be crudely divided into three distinct strands. One is the modeling of the decisions of individual generators based upon input risks (such as water flows for hydro generators). This considers risk from the perspective of a single generator, and does not consider market outcomes of multiple risk-averse generators, so is not particularly relevant to our model. Another strand takes the point of view of large consumers faced with a risky series of electricity prices. In this strand, prices are taken as exogenous, not formed through competition between risk-averse firms. The final strand, most relevant to this paper, is the study of risk-aversion in the formation of long-term contracts. Examples include articles by Powell (1993) and Neuhoff and De Vries (2004). Both of these papers use the expected utility method to model risk-aversion, since concave utility functions are known to mimic risk-averse behaviour. On the other hand, Rocha and Kuhn (2012) develop a multistage stochastic mean-variance optimization model to optimize portfolios of electricity derivative contracts for a single firm. Baldursson and von der Fehr (2007) analyze the behaviour of risk-averse firms in both wholesale and retail markets simultaneously. They consider a model where risk-averse firms vertically integrate, finding that vertical integration impairs market performance by increasing the gap between contract and (expected) spot prices. Their measure of risk is mostly the constant relative risk-aversion (CRRA) measure commonly used by economists.[2] Firms are price takers in the wholesale (spot) market, with either fixed retail prices, or retail prices that are linked to wholesale prices. (Retail prices are not determined by retail competition.) The goal of their paper is to see if long-term contracts can still enhance competition when risk-aversion is incorporated. CRRA has also been employed by Chronopoulos, Reyck, and Siddiqui (2014) in modeling entry and duopoly competition in a market setting where firms are risk averse against price uncertainty. Finally, Woo, Karimov, and Horowitz (2004) model how electricity distribution companies can purchase forward contracts in order to minimize expected future procurement costs, while constraining the Value at Risk (VaR) to be below a specified level.

CVaR has been previously employed to model generator behaviour in the operations research literature. Papers in this area include Carrion (2008) and Carrion, Gotzes, and Schultz (2009). Again, many of these papers model a single firm's point of view, considering the best response of the firm given a perceived risk. However, recent papers by Ralph and Smeers (2011) and Philpott, Ferris, and Wets (2015) present Nash games under risk, giving examples of equilibria where risk is modeled using CVaR. We also compute Nash equilibria with risk-averse agents, specifically considering the equilibrium responses of multiple firms acting to maximize their profits, less conditional value-at-risk. We demonstrate that the behaviour of risk-neutral firms compared to risk-averse firms can be very different, with obvious consequences for policy decisions.

We begin by briefly presenting the concept of conditional value-at-risk and discussing how we employ it in the context of electricity markets. We then construct a model of an electricity market with three stages: entry, retail competition to form long-term retail contracts, followed by wholesale market-clearing to set the spot prices.[3] We use this model first to present a simple example to illustrate the impact of risk-aversion on retail pricing and finally a general model formulation which we use to show how nodal price risk can cause generators to limit their retail obligations to nodes where they have their own generation capacity. We give examples that illustrate how risk-averse firms' behaviour varies significantly from their risk-neutral counterparts, and finish with an extended policy example: a two-node stylised analysis of a generator asset-swap in the New Zealand Electricity Market, designed to reduce risk and thereby decrease retail prices.

2. Modeling risk-averse firms

When companies are making investment decisions or entering contracts they do not solely consider the expected benefit from the decisions. They also take into account the consequences if the return on the investment is lower than expected – behaviour which is known as risk-aversion. Companies are responsible to shareholders over both the short and long term, so they need to make decisions that seek to minimize their risk, while at the same time maximizing expected profit.

In the finance literature, mean-risk optimization is commonly used in portfolio optimization. This approach involves solving a bi-objective optimization problem and typically results in finding a set of Pareto optimal solutions, known as the *efficient frontier*. In such models risk is typically measured by the variance of the return or by using the downside-risk, as introduced by Markowitz (1959).

Although firms often use the concept of value-at-risk to measure their risk exposure, VaR has not been used much in equilibrium models due to its intractability. In particular, utility functions incorporating value-at-risk may not be convex (Artzner, Delbaen, Eber, & Heath, 1999). In order to formalize the concept of risk, Artzner et al. (1999) introduced *coherent* measures of risk. A coherent risk measure must comply with the following four axioms: sub-additivity; translation invariance; positive homogeneity; and monotonicity. Risk measures complying with these axioms exhibit key properties that are valuable for a risk-averse agent. For example, sub-additivity ensures that there is a risk-pooling effect: the sum of risks is greater than or equal to the risk of the sum. Critically, none of the above risk measures are coherent. The value-at-risk measure, for example, violates the sub-additivity property.

Here we use *conditional value at risk* (CVaR)[4] that is a coherent risk measure. Recall that CVaR at level α of some random return z is simply the expected loss if one's interest were restricted to the lowest 100α percent of returns. If returns z are continuously distributed with some distribution function $F(z)$ and associated probability density function $f(z)$ then $\mathrm{CVaR}_\alpha(z)$ can be written as:

$$\mathrm{CVaR}_\alpha(z) = -\frac{1}{\alpha}\int_{-\infty}^{F^{-1}(\alpha)} z f(z) dz.$$

Rockafellar and Uryasev (2000) present a formulation of CVaR which enables the bottom 100α percent of outcomes to be computed through a linear program. Moreover, for profit functions that are concave in the decision variables, we have a convex optimization problem when we maximize a weighted combination of expected profit and risk. Below, θ is a parameter between 0 and 1 and changes the weightings on risk and return, z is a random variable which may be a function of some of the parameters.

$$\max \quad (1-\theta)\mathbb{E}[z] - \theta\mathrm{CVaR}_\alpha(z). \tag{1}$$

In this paper, we use the above mean-risk formulation (with risk measured using CVaR) in our model of an electricity retail market with risk-averse firms. See Shapiro, Dentcheva, and Ruszczynski (2009) for further details as to how CVaR can be incorporated into a convex optimization problem. In Section 5 we present a mathematical program that utilizes the above objective function.

3. Electricity market model with one node

In this section we begin our exposition by outlining the model in the single node case. Assume that electricity is traded in a market

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[2] They use a more general formulation in their initial modeling.

[3] We assume a competitive equilibrium in the wholesale market.

[4] Also known as average value at risk, or expected shortfall.