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Decision Support

A tractable interest rate model with explicit monetary policy rates☆

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ABSTRACT

This paper proposes a novel interest rate model that presents simple analytical pricing formulas for interest rate-based derivatives, including swaps, futures, swaptions, caps and floors. Exploring the regime-switching feature of Markov chains, the proposed model focuses on discrete changes in the central bank policy rates – the main driver of short-term rate fluctuations. An empirical analysis shows that the proposed model generally outperforms other standard short-term rate models in fitting cross-sections of options prices. Moreover, the explicit nature of policy rates, to some extent, enables the model to infer risk-neutral probabilities of the central-bank rate decisions.

1. Introduction

According to data collected by the Bank for International Settlements (2015), the notional amount outstanding of interest rate derivatives has been multiplied by eight over the last fifteen years. This amount is more than six times larger than world GDP in 2014, sixty times larger than that of equity-linked derivatives.

Any change in interest rates affects the market value of these vast amounts of derivatives as well as the pricing of newly contracted ones. As a result, the sources of interest rate movements are highly scrutinised. This contributes to the particular attention given to central banks by financial markets. Indeed, monetary authorities steer the short-end of the yield curve by deciding on the policy rates. Moreover, the influence of central bankers is not limited to short-term rates; theoretical and empirical evidence points to a strong impact of monetary policy decisions and communication on medium to long-term rates.[1]

Despite the pervasive relationship between monetary policy and the yield curve, very few term structure models explicitly incorporate central-bank policy rates. In particular, standard models, where short-term rates typically follow Gaussian or square-root diffusions, do not accommodate the fact that most of the short-term rate fluctuations are accounted for by infrequent changes in the policy rates. This notably implies that conditional distributions of future interest rates derived from these models are not fully realistic, at least for short-term interest rates.[2] Insofar as interest rate derivatives pricing precisely depends on these distributions, one may be concerned about the pricing ability of standard models. Nevertheless, the latter are widely used and various studies illustrate their reasonable fitting performances (e.g. Brigo & Mercurio, 2001, 2007; Gupta & Subrahmanyam, 2005; Martellini, Priaulet, & Priaulet, 2003). These comforting results are however obtained for calibration exercises performed on homogenous sets of market prices where, typically, a single class of interest rate options is present. The rare analyses that consider the ability of standard models to simultaneously price a wider variety of derivatives show less positive results (e.g. Jagannathan, Kaplin, & Sun, 2003). Arguably, a model based on more realistic interest rate dynamics may have less difficulties in matching large cross-sections of market prices. The results presented below do support this view.

In this paper, I introduce a novel interest rate model where policy rates occupy the central place. An extensive and original use of Markov-switching chains makes it possible to capture the step-like dynamics of policy rates. In spite of the important number of regimes it features – typically higher than 100 – the modeling approach remains particularly tractable, offering closed-form pricing formulas for swaps, futures, swaptions, caps and floors. These formulas in-

☆ An earlier version of this paper was circulated under the title: "Fixed-Income Pricing in a Non-Linear interest rate Model". A companion web-interface allows to reproduce several of the charts presented in this paper, this interface is available at: https://fixed-income.shinyapps.io/NLIR. This work has benefited from stimulating discussions with Ben Craig, Darrel Duffie, Refet Gürkaynak, Alain Monfort, Sarah Mouabbi and Jing Cynthia Wu. I thank participants at the 2nd International Workshop on "Financial Markets and Nonlinear Dynamics" and seminar participants at the Banque de France for helpful comments. A substantial part of this work was completed when the author was at the Banque de France. This paper expresses the views of the author only; they do not necessarily reflect those of past or present employers of the author.

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[1] See, among many others, Kuttner (2001), Rigobon and Sack (2004) Gurkaynak, Sack, and Swanson (2005) or Cochrane and Piazzesi (2005).

[2] Consider for instance the case where the policy rate is a multiple of 25 bps and where the overnight interbank rate is always close to – and on average equal to – the policy rate. Then, the conditional distribution of future overnight interest rate should be multimodal, with modes being multiple of 25 bps. This cannot be replicated by standard models.

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volve only basic algebraic operations, ensuring fast and simple model calibrations.[3]

Based on euro-area yield and option data covering the last decade, an empirical analysis explores the fitting performance of the model. Specifically, at each date in the sample, the model is calibrated so as to fit a cross-section of swap yields and interest rate option prices (swaptions, caps and floors). The resulting fit is compared with those obtained with traditional short-rate models.[4] The results suggest that the proposed model generally outperforms its competitors in fitting cross-sections of options prices. In a second exercise, the different models are calibrated on swaps and swaptions data only. Prices of caps and floors are then computed using the calibrated models. It appears that the cap and floor prices stemming from the regime-based model are closer to the (out-of-sample) observed prices than those derived from the other models. For instance, over the period 2010–2014, the mean absolute fitting error on cap and floor prices resulting from the present model is of 5 basis points – that is about 7 percent of the option prices – against 13 basis points – about 17 percent of the option prices – for the best of the alternative models. Hence, when it comes to price options for which no prices are available, the present model appears to be more reliable than its competitors; such a property makes the model particularly appealing for trading purposes.

In an additional application, I show how this framework stands as a relevant tool to recover market expectations of monetary policy – an application that is of particular interest for central bankers and market analysts alike. Being the first model in the literature that combines (a) explicit monetary policy rates and (b) a simple calibration based on derivatives prices, this model offers a natural and convenient way to recover market-implied probabilities of future policy changes. As illustrated by Krueger and Kuttner (1998) and Gurkaynak, Sack, and Swanson (2007), certain untransformed market data – such as the Fed funds futures rates or overnight indexed swaps (OISs) – can readily be interpreted as market expectations of future policy rates. Yet, while these rates may be informative to reveal the mean path expected by market participants, they do not reflect the market views regarding possible deviations from this path. Instead, as argued in Carlson, Craig, and Melick (2005) and Emmons, Lakdawala, and Neely (2006), option prices contain useful information about the market perception of uncertainty. The last two papers use specific options, namely the options on U.S. Fed funds futures, to recover the distributions of future policy rates. Unfortunately, such options do not trade in the euro area, as in many other currency areas.[5] By contrast, a wide range of standard interest rate derivatives can feed my model, allowing it to capture an aggregated market view.

The rest of the paper is organized as follows. Section 2 briefly reviews the literature of interest rate modeling. Section 3 sets forth the model and the pricing formulas. A basic specification of the model is explored in Section 4. Empirical exercises are presented in Section 5. Section 6 concludes.

2. Related literature

The framework introduced in this paper extends the family of so-called short-rate models. These models are defined through the dynamics – i.e. the transition distribution – of the short-term rate.[6] Once this dynamics is specified, the prices of all interest rate-sensitive instruments result from the computation of the conditional expectation of discounted payoffs (see for instance Cochrane (2001), Sundaram (1997) or Shreve (2004)). The most prominent short-rate models include: the one-factor Gaussian model of Vasicek (1977), its two-factor version of Hull and White (1994) and the square-root model of Cox, Ingersoll, and Ross (1985) (CIR hereinafter). Important extensions of these models have been proposed by Hull and White (1990), Jamshidian (1995), Brigo and Mercurio (2001) and Moreno and Platania (2015). These models feature closed-form formulas for a variety of interest rate derivatives, which explains their popularity among practitioners and researchers alike.

The model I introduce below differs from the previous ones in that it explicitly incorporates policy rates. This paper provides evidence that this novel model is better at fitting large cross-sections of derivative prices. Piazzesi (2005) and Fontaine (2014, Chap. 9) also introduce term-structure models with explicit policy rates. Although the last two frameworks allow for a relatively simple computation of model-implied long-term yields, they do not offer closed-form formulas to price interest rate options. An additional drawback of these two models is that they cannot be made consistent with the existence of a lower bound for nominal yields; this may bias the pricing of interest rate instruments in a low-interest rate environment such as the one prevailing at the time of writing.[7]

Another important class of interest rate models used for derivative pricing is that of *LIBOR market models*. These frameworks, originally due to Brace, Gatarek, and Musiela (1997) and Miltersen, Sandmann, and Sondermann (1997), have the great advantage of being consistent with Black (1976)'s formula, that is the standard formula employed in the interest rate options market. The tractability of these models is however limited to the pricing of a pre-defined class of interest rate options (cap/floor options versus swaptions).[8] By contrast, the model proposed in this paper offers closed-form formulas for all considered interest rate instruments and derivatives.

This paper also relates to the strand of the literature investigating the dynamics of policy rates (Balduzzi, Bertola, & Foresi, 1997; Balduzzi, Bertola, Foresi, & Klapper, 1998; Hamilton & Jorda, 2002; Hu & Phillips, 2004; Rudebusch, 1995). However, this literature focuses on the very-short-term rates and the models it employs are not suited to price long-term bonds or interest rate derivatives (unless resorting to time-demanding Monte-Carlo simulations).

3. Model

3.1. Dynamics of the short-term rate

This section presents the risk-neutral dynamics followed by the overnight interbank rate r_t, and its resulting implications in terms of pricing.[9]

The short-term rate is split into two components:

$$r_t = \Delta' z_t + \xi_t, \tag{1}$$

[3] By incorporating policy rates, my model features more structure than other short-rate models. However, as the latter, it does not account for other factors, such as demand pressure for specific instruments, that may also affect the values of derivatives. This is beyond the scope of this paper.

[4] The alternative models are the Vasicek model, the CIR model, the extended CIR model (CIR++) and the extended two-factor Gaussian model. An in-depth description of these models can be found in Brigo and Mercurio (2007).

[5] Besides, as mentioned by Emmons, Lakdawala, and Neely (2006), these futures-option contracts do not have sufficient liquidity to derive expectations more than a few months ahead.

[6] Comprehensive reviews of the literature on interest rate modeling are provided by Boero and Torricelli (1996), Broadie and Detemple (2004) and Schmidt (2011); influential books are those of Rebonato (1998), Shreve (2004) and Brigo and Mercurio (2007).

[7] For instance, a model that does not preclude negative interest rates produces strictly positive price for all interest rate floors, even those with strongly negative strikes. The consistency of my model with a lower bound for nominal interest rates is further discussed in Section 5.6.1.

[8] That is, depending on model assumptions, LIBOR models can provide closed-form formulas for (a) caps and floors or for (b) swaptions, but not simultaneously (see Rebonato and Pogudin, 2011 or Brigo and Mercurio, 2007, Chap. 6).

[9] In a general discrete-time finite-horizon setting, the existence of a risk-neutral measure is equivalent to the absence of arbitrage opportunities (Delbaen & Schachermayer, 1994).