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Decision Support

A technique to develop simplified and linearised models of complex dynamic supply chain systems

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ABSTRACT

There is a need to identify and categorise different types of nonlinearities that commonly appear in supply chain dynamics models, as well as establishing suitable methods for linearising and analysing each type of nonlinearity. In this paper simplification methods to reduce model complexity and to assist in gaining system dynamics insights are suggested. Hence, an outcome is the development of more accurate simplified linear representations of complex nonlinear supply chain models.

We use the highly cited Forrester production-distribution model as a benchmark supply chain system to study nonlinear control structures and apply appropriate analytical control theory methods. We then compare performances of the linearised model with numerical solutions of the original nonlinear model and with other previous research on the same model.

Findings suggest that more accurate linear approximations can be found. These simplified and linearised models enhance the understanding of the system dynamics and transient responses, especially for inventory and shipment responses.

A systematic method is provided for the rigorous analysis and design of nonlinear supply chain dynamics models, especially when overly simplistic linear relationship assumptions are not possible or appropriate. This is a precursor to robust control system optimisation.

1. Introduction

In supply chains, the variability in the ordering patterns often increases as one moves up the chain, towards the factory and the suppliers (Dejonckheere, Disney, Lambrecht, & Towill, 2003). This variance is called the bullwhip effect, "one of the most widely investigated phenomena in supply chain management" (Chatfield & Pritchard, 2013). Even to this day the seminal works of Forrester (1958, 1961), that formed the foundation for System Dynamics, are almost always referred to synonymously with the bullwhip effect (e.g. Chatfield & Pritchard, 2013; Zhang & Burke, 2011). In citing Forrester's works authors refer to the original production-distribution model, which may now be classified as a representation of a supply chain, as a bullwhip generator archetype and the use of System Dynamics simulation as a technique for exploring opportunities to mitigate unwanted dynamic behaviour.

While System Dynamics simulation is often used in the analysis and redesign of supply chain models that exhibit nonlinearities, quantitative analytical approaches are more often restricted to linear representations of supply chains. Hence, much of the research on supply chain dynamics either takes a 'trial and error', experimental, simulation approach to redesign (Forrester, 1961; Poles, 2013; Shukla, Naim, & Yaseen, 2009; Spiegler & Naim, 2014; Sterman, 1989) or develops exact solutions of models that are already linearised approximations to the real-world situation (Disney & Towill, 2005; Gaalman & Disney, 2009; John, Naim, & Towill, 1994; Towill, 1982; Zhou, Disney, & Towill, 2010).

While the original Forrester supply chain model is often quoted as the embodiment of the bullwhip effect it has had little exposure with respect to its use as a benchmark for applying supply chain analysis and redesign methods, with the notable exceptions of Wikner, Naim, and Towill (1992) and Jeong, Oh, and Kim (2000). The former explore a simplification approach to understanding the causes of the bullwhip effect (Wikner et al., 1992), while the

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[1] This article is dedicated to the work, friendship and memory of Professor Denis R. Towill.

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latter apply a linearisation approach but with an analysis totally reliant on simulation. Analytical tools to link system dynamics model structures to different system modes of behaviour have recently become available and explored for linear models (Saleh, Oliva, Kampmann, & Davidsen, 2010). However, there is still a need to expand the existing body of knowledge regarding robust control of nonlinear supply chains. "Nonlinearity can introduce unexpected behaviour in a system" (Forrester, 1961), causing instability and uncertainty and therefore needing to be rigorously analysed.

Our paper aims to present a technique to develop both simplified and linearised models of complex, nonlinear supply chain systems. We seek to gain greater insights into the underlying mechanisms that create supply chain dynamics and to provide guidelines for undertaking system dynamics simulation in a time effective and productive way. The simplified and linearised form may also be a precursor to robust optimisation of nonlinear decision rules in supply chains, a gap in the existing body of knowledge due to the complexities of dealing with seemingly intractable mathematics. We utilise the original Forrester model as a benchmark, as per Wikner et al. (1992) and Jeong et al. (2000), given it is a complex representation of a production-inventory control system with nonlinearities and it is highly cited for describing the behavioural dynamics of supply chains. The currency of the model is also evidenced by 333 citations related to supply chains in 2013–2014 (according to Google Scholar searched on 27 August 2014) and by Singhal and Singhal (2012) noting that Forrester's papers still represent well the real-world phenomenon of fluctuations and oscillations since it contains a combination of simulated data and case study data to examine the flow of materials and information in a supply chain.

In summary, we aim to determine the methodological benefits of nonlinear control theory in supporting simulation based research on supply chain dynamics studies. This research is particularly relevant to operations research scholars exploring nonlinear dynamic systems. Moreover, future applications of the proposed technique may also benefit practitioners in improving supply chain performance. As Ivanov and Sokolov (2013) pointed out "useful tools for quantitative analysis of control and systems theory for a wide supply chain management research community remain undiscovered". Our work addresses this shortcoming.

2. Nonlinear system dynamics

A nonlinear system is one whose performance does not obey the principal of superposition. This means that the output of a nonlinear system is not directly proportional to the input and the variables to be solved cannot be expressed as a linear combination of the independent parts (Atherton, 1975). In this section, we briefly review methods for analysing nonlinear system dynamics and highlight where certain methods have already been used in supply chain dynamics research.

When confronted with a nonlinear system the primary approach utilised by system designers is to identify an equivalent linear representation. A justification for this is that there are a variety of analytical techniques available in linear control theory that are not so readily applicable in the analysis of nonlinear systems. While linear control theory is well established, the literature lacks a unique nonlinear theory that strives for generality and applicability (Hotz & Vogel, 2014; Rugh, 2002).

The lack of generality, coupled with often indefinite research methods, has led to a confusion of terminologies making it a challenge to determine a listing of all existing techniques and their applicability in the analysis of nonlinear feedback systems. Table 1 lists those methods that have been sufficiently acknowledged in the literature. The table highlights the type of nonlinearity that each method addresses, the assumptions or limitations that need due consideration in their application and citations of where they have been applied in a supply chain dynamics context. The choice of each method may also depend on the degree of complexity involved in the setting up of a mathematical model, the type of data available for analysis and the analytical skills of the researcher or supply chain designer.

Most research on nonlinear supply chain systems has been undertaken via simulation methods. Table 1 gives only a small number of examples from a plethora of papers that utilise simulation to analyse nonlinearities in supply chains. This research has led to the understanding of particular phenomena; such as:

- Stability and chaos (Larsen, Morecroft, & Thomsen, 1999; Laugesen & Mosekilde, 2006),
- The impact of capacity and batching constraints (e.g. Cannella, Ciancimino, & Márquez, 2008; Hamdouch, 2011; Ivanov, Hartl, Dolgui, Pavlov, & Sokolov, 2014; Juntunen & Juga, 2009; Paik & Bagchi, 2007), inaccuracies in inventory (Cannella, Framinan, Bruccoleri, Barbosa-Póvoa, & Relvas, 2015), reverse logistics (Turrisi, Bruccoleri, & Cannella, 2013) and collaborative strategies (e.g. Cannella & Ciancimino, 2010; Spiegler & Naim, 2014) on system dynamics and supply chain performance,
- Bullwhip effect in service supply chains Akkermans and Voss (2013)
- Shipment planning (Mula, Campuzano-Bolari, Diaz-Madronero, & Carpio, 2013; Shukla et al., 2009) and
- The effects of psychological pressure, misperceptions and misjudgement in work environments (Bruccoleri, Cannella, & Porta, 2014; Sterman, 1989; Syntetos, Georgantzas, Boylan, & Dangerfield, 2011).

In contrast, there is limited research on the use of analytical methods. Many of the analytical studies on nonlinear system dynamics were undertaken in the same decade Forrester launched the World Dynamics model (Forrester, 1971), which is a simpler model when comparing to the production-distribution model. Cuypers (1973) used averaging techniques for linearising discontinuous nonlinearities in the World Dynamics model. One year later, numerical perturbation techniques and model simplification, involving the removal of variables with little variation, were also explored (Cuypers & Rademaker, 1974). Ratnatunga and Sharp (1976) proposed the use of numerical analysis to linearise and reduce orders of system assuming that nonlinear associations can be approximated to a first order function. Mohapatra (1980) identified and categorised different types of nonlinearities in business system dynamics research. Although his work recommends a number of techniques to deal with nonlinearities, there is no implementation of such methods in the paper.

Within a supply chain management context, Wikner et al. (1992) undertook in-depth analysis of the complex Forrester production-distribution model (1961). By using averaging techniques and block diagram manipulation, they linearised and simplified the original model and provided more qualitative analytical insights. For example, they highlighted the lack of feedback information fed into the manufacturing rate and the separation of 'real' and 'safety' orders. By following the same simplification and linearisation steps, Naim, Wikner, Towill, and Marques (2012) achieved the same result for the discrete z-domain model. In contrast, instead of using an averaging technique, Jeong et al. (2000) used small perturbation theory to linearise the continuous nonlinearities in the Forrester model and Matsubara's time delay theorem to obtain a first-order delay approximation to represent an upstream echelon.

Another advocate of the use of analytical methods rather than just the use of exhaustive repeated simulation are Saleh et al. (2010). They suggest the use of small perturbation theory to