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Complexity of routing problems with release dates

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ABSTRACT

In this paper we consider a routing problem where uncapacitated vehicles are loaded with goods, requested by the customers, that arrive at the depot over time. The arrival time of a product at the depot is called its release date. We consider two variants of the problem. In the first one a deadline to complete the distribution is given and the total distance traveled is minimized. In the second variant no deadline is given and the total time needed to complete the distribution is minimized. While both variants are in general NP-hard, we show that they can be solved in polynomial time if the underlying graph has a special structure.

1. Introduction

Many carriers consolidate distribution by introducing a distribution center in the vicinity of final customers. Goods that are delivered to the centers usually by trucks are unloaded, sorted, consolidated and delivered to final customers by, usually smaller, trucks that perform routes, each visiting several final customers. A typical example of such distribution system is encountered in fast parcel delivery. Other examples are found in less-than-truckload distribution systems. Also, in the context of city logistics, often goods are consolidated and distribution in the city center is carried out by means of electric vehicles.

Usually, the phase of receiving goods arriving to these centers and the distribution to final customers are decoupled and treated independently from each other. When trucks reach the center, goods are received and temporarily stocked. At a later time, deliveries to final customers are organized. Goods are picked up, loaded into trucks and distributed. The two phases can be handled separately because of inventory that is built in the centers. Due to the pressure towards cost reduction in both inventory and transportation management, third party logistics have grown in number and size. The management of distribution centers is becoming more and more dynamic. Goods arrive at any time during the day, and routes are continuously organized and started for delivery.

The classical vehicle routing problems (see the recent book by Toth & Vigo, 2014) consider situations where the goods to be distributed are all available at the depot at the time the distribution starts. This is true also when time is considered such as in the problems with time windows (see Desaulniers, Madsen, & Ropke, 2014), in the periodic vehicle routing problems (see Francis, Smilowitz, & Tzur, 2008) and in inventory routing problems (see Bertazzi & Speranza, 2013; Coelho, Cordeau, & Laporte, 2013). A vehicle routing problem taking into account time issues recently introduced in the literature is the multi-period vehicle routing problem with due dates (see Archetti, Jabali, & Speranza, 2015) where goods have to be delivered within a certain time period. Also, city logistic problems deals with the distribution of freight over a planning horizon where different means of transportation have to be synchronized (see Crainic, Ricciardi, & Storchi, 2009).

Taking into account the arrival times of goods means considering a more dynamic organization of the distribution. In this case one has to organize the delivery routes with the additional constraint that goods are not all available at the depot at the start of the distribution. The routing problems must consider the additional issue of whether it is better to wait for additional goods to arrive and have a better loaded vehicle, or to start a route of the vehicle with the currently available goods. In the following, we call the arrival time of a product at the depot its *release date*. Problems with release dates, inspired by real applications, are introduced in Cattaruzza, Absi, and Feillet (2014); Cattaruzza, Absi, Feillet, Guyon, and Libeaut (2013).

Most of the classical vehicle routing problems can be extended to consider release dates. We call this new class Vehicle Routing Problems with release dates (VRP-rd). The classical performance measure in vehicle routing problems is the distance. When release dates are

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considered, the minimization of the distance is achieved through waiting for all goods to arrive at the depot and solving a classical routing problem. This, however, may mean that the time required to complete the distribution is longer than desired. The completion time becomes a different relevant performance measure. Practical motivations for minimizing the completion time are the willingness to make the drivers available as early as possible for further tasks, the goal of improving the customer service or of avoiding rush hours. Motivations to minimize the distance are more classical and include the minimization of operational costs and related measures, such as pollution.

In this paper we focus on the novelty implied by the presence of release dates and analyze basic problems in the class of VRP-rd where vehicles are uncapacitated (the generalization to capacitated vehicles will be mentioned in the conclusions). As commonly done in routing problems, we assume that the distance traveled coincides with the traveling time. We consider the case where a single vehicle is allowed to perform several trips during the time horizon (say, the day), one after the other, and the case where a fleet of vehicles is limited to a single route each. We consider two different objectives, the minimization of the total traveling distance within a deadline, and the minimization of the completion time, that is the sum the maximum value over all vehicle routes of the sum of traveling time and waiting time. We investigate the computational complexity of the optimization problems when the graph describing the locations of depot and customers has special structures, namely is a star or a line. The star models in an abstract way situations where the depot is at the center of the distribution area, whereas the line situations where customers are all located along a road. These special structures allow us to explore characteristics and properties implied by the presence of the release dates. We will show that, while the problems are in general NP-hard, they can be solved in polynomial time on stars and lines.

The paper is organized as follows. The following section provides the problem definition, summarizes the problem variants we study and summarizes the main contributions of the paper. Section 3 focuses on the Traveling Salesman Problem with release dates while Section 4 deals with the Uncapacitated Vehicle Routing Problem with release dates. Finally, conclusions are drawn in Section 5.

2. The vehicle routing problem with release dates

Let $G = (V, A)$ be a complete graph. A traveling time and a traveling distance are associated with each arc $(i, j) \in A$. For the sake of simplicity, these two values are assumed identical and are denoted by t_{ij}. It is also assumed that the triangle inequality is satisfied. The set of vertices V is composed by vertex 0 which identifies the depot and the set N of customers, with $|N| = n$. The release date for customer $i \in N$ is denoted by r_i, $r_i \geq 0$. This means that the goods for customer i arrive to the depot at time r_i. If goods for customer i are already at the depot at the beginning of the time horizon, because they arrived overnight or earlier, then $r_i = 0$. We consider two different objectives. First, a deadline to complete the distribution is given and we aim at minimizing the total traveling distance. Second, no deadline is given and we aim at minimizing the total time needed to complete the distribution.

We consider two special cases: a case where a single vehicle is allowed to perform several trips during the time horizon (say, the day), one after the other, and a case where an unlimited fleet of vehicles is limited to a single route each. In both cases, capacity constraints are not considered. We call the first case the *Traveling Salesman Problem with release dates* (TSP-rd) while the second is called the *Uncapacitated Vehicle Routing Problem with release dates* (UVRP-rd). For each case we consider the two above mentioned variants, where in the first variant the objective is to minimize the total traveling distance in such a way that the distribution is completed within a deadline T. We will refer to these problems as TSP-rd(distance) and UVRP-rd(distance), respectively. In the second variant we minimize the completion time, that is the maximum value over all vehicle routes of the sum of traveling time and waiting time of the vehicle. We refer to these problems as TSP-rd(time) and UVRP-rd(time), respectively. Note that, since we assume that the traveling time and the traveling distance associated with each arc (i, j) are identical, the objective function of the TSP-rd(distance) and the UVRP-rd(distance) corresponds to the minimization of the sum of t_{ij} of the arcs traversed. When considering the TSP-rd(time) and the UVRP-rd(time), the objective function minimizes the maximum value over all vehicle routes of the sum of t_{ij} of the arcs traversed plus the waiting time at the depot. The waiting time is due to the fact that a vehicle has to wait at the depot until the latest release date of a customer that it is going to serve.

Table 1
Polynomial algorithms on the star.

TSP-rd(time)	Visit in the order of release dates
TSP-rd(distance)	Constant value
UVRP-rd(time)	One customer per vehicle
UVRP-rd(distance)	Constant value

Table 2
Polynomial algorithms on the line.

TSP-rd(time)	See algorithm in Section 3.2.2
TSP-rd(distance)	See algorithm in Section 3.2.3
UVRP-rd(time)	One customer per vehicle
UVRP-rd(distance)	See algorithm in Section 4.1

We focus on the study of the above mentioned problems on special graphs, namely the line and the star. The main contribution of this paper is to show that all the studied variants are polynomially solvable on these two special graphs. While the proof is straightforward for some of the cases considered, it is not for the case of the TSP-rd on the line. Tables 1 and 2 summarize the results that we present in the following sections. We provide a short hint about how the optimal solution is built when the solution of the problem is straightforward, while we refer to the proof provided in the related section when it is not.

3. The traveling salesman problem with release dates

In the *Traveling Salesman Problem with release dates* (TSP-rd) a single vehicle is considered. We study the TSP-rd(distance) and the TSP-rd(time). For both variants, if all release dates are equal to zero, due to the triangle inequality, it is optimal to load all goods on the vehicle and perform a single route. In this case the TSP-rd is equivalent to the TSP. This also implies that the TSP-rd is NP-hard, having the TSP as a special case. Note that if in the general case the deadline is very large, the TSP-rd(distance) becomes equivalent to the TSP, as it is optimal to wait until all goods are available and perform a single TSP tour. This is not true for the TSP-rd(time).

TSP-rd(time) always admits feasible solutions. For the TSP-rd(distance) to have a feasible solution, it must be possible to serve each customer within the deadline, that is a necessary feasibility condition is that $r_i + 2t_{0i} \leq T,\ i = 1, \ldots, n$. It is not trivial, however, to guarantee that a feasible solution exists as it depends on the complete set of release dates and on the traveling times. Consider, for example, two customers, with release dates equal to zero. The traveling time from the depot to each customer is equal to 4 and the traveling time between them is equal to 2. Suppose the deadline is 8. It is feasible to serve each customer individually but not to serve both within the deadline.

In the following we consider the special cases of the TSP-rd on the star and on the line. While the solution of both variants of the TSP-rd