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Decision Support

Product differentiation and entry timing in a continuous time spatial competition model[☆]

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ABSTRACT

We extend the well-known spatial competition model (d'Aspremont, Gabszewicz & Thisse, 1979) to a continuous time model in which two firms compete in each instance. Our focus is on the entry timing decisions of firms and their optimal locations. We demonstrate that the leader has an incentive to locate closer to the center to delay the follower's entry, leading to a non-maximum differentiation outcome. We also investigate how exogenous parameters affect the leader's location and firms' values and, in particular, numerically show that the profit of the leader changes non-monotonically with an increase in the transport cost parameter.

1. Introduction

Researchers in economics and marketing have emphasized the importance of (horizontal) product differentiation in the context of firm competition (e.g. Brown, 1989; d'Aspremont, Gabszewicz, & Thisse, 1979; Lancaster, 1990). When firms launch their new products into markets, timing and product characteristics are some of the important factors for their profits (e.g. Krishnan & Ulrich, 2001). Taking into account firms' decisions regarding product differentiation, researchers theoretically and/or empirically investigate how firms determine the timing of launching their products and those characteristics (e.g. Lambertini, 1997; Thomadsen, 2007).

From the theoretical point of view, Lambertini (2002) presented pioneering work that discusses the strategic interaction between the optimal locations of the inventor (the market leader), who anticipates subsequent entry and the location choice of the follower in a Hoteling-type spatial competition model, as in d'Aspremont et al. (1979).[3] He was the first to introduce a dynamic model in the sense that time is continuous, the firm locations are fixed once entry is made and that firms earn their profits in each instance. Regarding the time structure, several papers deal with sequential locational entry in a discrete time model, which allows qualitative analyses such as how many steps the timing of investment would change given a change in other parameters (e.g. Prescott & Visscher, 1977). However, a more rigorous quantitative analysis, such as determining the percentage change in the investment time attributable to a percentage change in a parameter, requires a continuous time model.[4]

This novel point is from Lambertini (2002) and differs significantly from those in related theoretical papers discussing sequential

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3 Location point is interpreted as a firm's differentiation selection because the distance between a firm's location point and a consumer's address corresponds to that between a firm's attribute and a consumer's ideal point. This interpretation is standard in spatial economics and marketing literature.

4 Continuous time models are often used in models such as real option game models that investigate the timing problem of firms' entry without the locational context (e.g. Dixit & Pindyck, 1994, Chapter 9; Azevedo & Paxson, 2014). These studies introduce one or more probabilistic fluctuations into their models. Our model is deterministic and does not focus on this randomness, but instead, focuses on the relation between location and entry timing. The real option game approach is useful for taking into account the endogenous timing decision.

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location choices based on Hoteling-type spatial competition models (e.g. Götz, 2005; Neven, 1987).[5] Those related papers are static Hoteling models in the sense that each firm has only one profit earning chance.[6] Lambertini (2002) considered two scenarios: (i) the follower's timing of entry is exogenous and (ii) the follower's timing of entry is probabilistically determined. Therefore, the follower does not endogenously determine its optimal timing of entry in either scenario. To summarize, Lambertini (2002) considered a continuous *time* model, but an endogenous entry *timing* model with continuous time *has not been considered* in locational models. Because the entry timing of followers significantly influences market leaders as well as followers (Kalyanaram, Robinson, & Urban, 1995; Vakratsas, Rao, & Kalyanaram, 2003), we need to overcome the weakness in the model given by Lambertini (2002) and endogenize the follower's entry-timing decision. Therefore, our paper substantially extends the model of Lambertini (2002).

We incorporate several aspects into the standard Hoteling duopoly model in d'Aspremont et al. (1979). The time horizon is infinite, as in Lambertini (2002). Each firm sets a price and earns a profit in each instance if it exists in the market, implying that a delay of entry causes a loss of profit opportunity. In anticipation of subsequent entry by the follower, the market leader initially sets its location. Because the leader's location decision influences the profits of the follower, it also affects the timing of the entry (the length of the monopoly period), thus representing an additional value of our paper. After the location choice of the market leader, the follower determines the timing of entry and its location. When the follower enters the market, it incurs an investment cost that exponentially decreases with the standard discount rate. In contrast, consumer size increases with a growth rate lower than the discount rate. By balancing the benefit and cost of staying outside, the follower determines its entry timing and location. We also note that this formulation is suitable for perishable goods as consumers repeatedly purchase the good.[7]

Compared with Lambertini (2002), our contributions in this paper are threefold. The first contribution is that we endogenize the follower's timing. The second contribution follows the first, as we introduce investment costs and a growth rate in consumer size to make the model more realistic. In addition, the growth rate ensures that the entry occurs within a finite time[8] and, in turn, affects the leader's location. The third contribution is a strategic interaction between the leader's location and the follower's entry timing. In addition to the effects considered by Lambertini (2002), the leader's moving closer to the center increases the follower's incentive to delay its entry, prolonging the monopoly regime. Thus, by endogenizing the follower's timing, the leader has a stronger incentive to move closer to the central point. Although this strategic interaction among the leader's location, the follower's location and its entry timing is an important aspect of this problem, Lambertini (2002) does not take into account this strategic interaction because of his assumption of an exogenous entry timing by the follower.

We also show that the follower always chooses to maximize the distance between the firms whereas the leader has an incentive to locate closer to the center to delay the follower's entry, possibly leading to a non-maximum differentiation outcome. Furthermore, the location interval between the leader and the follower is negatively correlated with the length of time for which the follower stays outside. These results are similar to those in Lambertini (2002), although the mechanism behind these results definitely differs between the two papers.

Finally, we show that the entry timing becomes earlier as the growth rate of consumer size or the parameter of consumer transport cost increases, and becomes later as the discount rate increases. We numerically investigate how those exogenous parameters influence their profits. A notable result is that the profit of the market leader non-monotonically changes with an increase in the consumer transport cost parameter.[9]

2. The model

Two firms, $i \in \{1, 2\}$, produce homogeneous goods. Consumers are uniformly distributed over the unit segment $[0, 1]$ as proposed by Hoteling (1929).[10] Each consumer at point $x \in [0, 1]$ repeatedly purchases at each instance $[t, t + dt)$ at most one unit of the good and decides from which firm to purchase if he does make a purchase.[11] The consumer at point $x \in [0, 1]$ incurs a quadratic transportation cost $c(x_i - x)^2$ and pays price p_{it} at time $t \in [0, \infty)$ when buying a good from firm i located at $x_i \in [0, 1]$. To summarize, the utility of the consumer at point $x \in [0, 1]$ at time $t \in [0, \infty)$ is given by

$$u_t(x; x_1, x_2, p_{1t}, p_{2t}) = \begin{cases} \bar{u} - p_{1t} - c(x_1 - x)^2 & \text{if purchased from firm 1,} \\ \bar{u} - p_{2t} - c(x_2 - x)^2 & \text{if purchased from firm 2,} \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where $\bar{u}$ denotes the gross surplus that a consumer at point x enjoys from purchasing the good, and c is a parameter describing the level of transportation cost or product differentiation. Let us assume that $\bar{u}$ is so large that each consumer prefers to purchase one good over not buying when at least one firm is present in the market.[12]

Assumption 1. $\bar{u} > 3c$.

The game proceeds as follows: each firm i chooses the time of entry $T_i \in [0, \infty)$ and location $x_i \in [0, 1]$ at the same time, and then chooses price $p_{it} : \Re_+ \to \Re_+$ at each time t, which is a function from time $t \in [0, \infty)$ to a real number $[0, \infty)$ and is displayed as p_{it} for simplicity. In addition, we assume that firm 1 is the leader who just entered at $T_1 = 0$, whereas firm 2 is the follower who enters at time T_2, to be subsequently and endogenously determined. In this way, firm 1 decides x_1 at time $T_1 = 0$ once and subsequently chooses price p_{1t} at each time t. After observing firm 1's actions before firm 2's entry, firm 2 chooses to enter at time T_2 and location x_2 and thereafter chooses p_{2t} at each time t. Firm i can choose its location only when it makes its entry in the project, at which time it incurs an entry cost $F_i(T_i)$. We also assume (without loss of generality) that $x_1 \le 1/2$ holds in equilibrium.

Now, let us describe the present value of the firms at time 0 given that firm 2 would enter at point x_2 at time $t = T_2$. Note that firm 1 enters at point x_1 at time $t = T_1 = 0$. The timing is exogenous[13] but x_1

[5] Many papers discussed sequential location choices in spatial competition models. Kress and Pesch (2012) and Biscaia and Mota (2013) provided comprehensive surveys on spatial competition.

[6] Lambertini (1997) and Meza and Tombak (2009) considered the endogenous timing of locations in such static Hoteling models.

[7] Perishable goods are defined as non-durable goods that last only for each infinitesimal instance of time. We will mention this point further in Discussion and concluding remarks.

[8] This phenomenon implies that just introducing a timing endogeneity into Lambertini (2002) without a growth rate yields no entry and a perpetual monopoly by the leader.

[9] The transport cost parameter can be interpreted as a parameter that describes the level of product differentiation because the cost parameter corresponds to a consumer's disutility between the consumer's ideal point and the degree of a product's attribute. This interpretation is standard in the literature on spatial economics and marketing.

[10] This setting and the following assumptions are standard in the literature on spatial economics.

[11] All consumers will respectively purchase a unit of product in equilibrium due to Assumption 1.

[12] In other words, firm 1, located at $x_1 = 0$, has an incentive to supply a positive amount at location 1, after maximizing its profit.

[13] A similar interpretation is made in Chronopoulos, De Reyck, and Siddiqui (2014) as a non-preemptive duopoly. In their paper, the roles of the leader and the follower are defined exogenously. Consequently, the future cash flows of the leader are discounted to time $t = 0$.