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European Journal of Operational Research 170 (2006) 72–90
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

A fuzzy stochastic single-period model for cash management

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Received 20 March 2003; accepted 18 June 2004
Available online 13 September 2004

Abstract

The major purpose of this paper is to apply a stochastic single-period inventory management approach to analyze optimal cash management policies with fuzzy cash demand based on fuzzy integral method so that total cost is minimized. We will find that, after defuzzification, the cash-raising amounts and the total costs between the fuzzy case and the crisp case are slightly different when the variation of cash demand is small. As a result, we point out that the fuzzy stochastic single-period model is one extension of the crisp models. In any case, one may conclude that a conscientious analysis in fuzzy mathematics like that presented in this paper provides a financial decision maker with a deeper insight into the more real cash management problem.

Keywords: Fuzzy sets; Stochastic single-period model; Cash management; Fuzzy integral; Signed distance method

1. Introduction

In real business environments, most financial managers have to determine how much cash to raise for normal day-to-day disbursement or protecting against unanticipated variations from budgeted cash flows in a business cycle, and furthermore to achieve the objective of minimizing expected total cost. Because various types of uncertainties and imprecision are inherent in the environment of cash management, they are classically modeled using the probability theory and therefore unpredictable cash demand is usually regarded as a random variable (D_{ran}) with a p.d.f. ($f(D)$), where $f(D)$ may be estimated by past statistical

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0377-2217/$ - see front matter
doi:10.1016/j.ejor.2004.06.017

data. In realistic situations, however, such estimation is often biased. For example, if big news or shocks occur in the financial market, cash demand for the next business cycle will show an unexpected fluctuation. Therefore, facing the dilemma of shortage or excess, the financial manager must adjust cash balances in accordance with real cash demand and reduce cash tied up unnecessarily in the system without diminishing profit or increasing risk.

The problem of managing cash balance is similar to that of managing physical inventory. Baumol (1952) first applied the EOQ model of inventory management in establishing a target cash balance. However, the Baumol model oversimplified the problem. Most importantly, it assumes that cash inflows and outflows are relatively stable and predictable, and it does not take into account any seasonal or cyclical trends. In the literature after Baumol, cash flow is usually regarded as a prescribed constant or a stochastic variable with time or raising quantity dependence (Tobin, 1958; Miller and Orr, 1966; Marquis and Witte, 1989). Recent studies relating certain storage systems to cash flow management have attracted much attention. Harrison et al. (1983) modeled the cash fund as a Brownian motion reflected at the origin. Harrison and Taksar (1983) considered impulse control policies: When the cash fund is too large, the controller may choose to convert some of his cash into securities; when the amount of cash decreases below some limit level, securities are reconverted into cash. Browne (1995) considered a firm with an uncontrollable cash flow and the possibility of investing in risky stock. In the study of Milne and Robertson (1996), a firm's cash flow is determined by a diffusion process and faces liquidation if the internal cash balance falls below some threshold value. Asmussen and Taksar (1997) and Asmussen and Perry (1998) provided jump diffusion models motivated by finance and general storage applications. Perry (1997) also extended the model of Harrison et al. (1983) by taking into account holding cost and unsatisfied demand cost functions to consider drift control for a two-sided reflected Brownian motion. Nevertheless, so far as we know, the optimal cash management policy for business using the concept of fuzzy cash demand has not been considered.

The cash management problem discussed in this paper is closely related to the single-period stochastic inventory, or "newsvendor," problem, which is a standard problem in the literature of inventory (Johnson and Montgomery, 1974; Hamidi-Noori and Bell, 1982). In such a problem, the management has to set the inventory at the level in which the value of the cumulative distribution function is equal to the cost/price ratio. Differing from previous studies, this paper attempts to develop a fuzzy model that takes the vague cash demand into account in order to provide a useful starting point for establishing a target cash balance in a fuzzy environment. We apply a stochastic single-period inventory management approach to analyze optimal cash balance with the considerations of fuzzy information and random components for cash demand (i.e. hybrid cash demand) so that total cost is minimized.

The rest of this paper is organized as follows. Section 2 states the preliminaries where we define fuzzy integral in Property 4, and employ the signed distance method similar to Yao and Wu (2000) to formulate the single-period model.

In Section 3, fuzzy integral method is employed to establish our fuzzy stochastic single-period model with regard to the cash management issue. After defuzzification, we can obtain the estimated total cost in the fuzzy sense in Formula 1. In Section 3.4, we estimate the fuzzy total cost by using exponential distribution as an example of Formula 1 shown in Theorems 2 and 3. In Section 4, we compare the result obtained from the fuzzy case in Section 3.4 to that of the crisp case with numerical operations. Finally, some characteristics of this model are discussed in Section 5 and concluding remarks are in Section 6.

2. Preliminaries

In order to apply the signed distance and the fuzzy integral method to formulate our problem, the following definitions are provided with some relevant operations.