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Discrete Optimization

The multiple vehicle pickup and delivery problem with LIFO constraints

Enrique Benavent[a], Mercedes Landete[b], Enrique Mota[a], Gregorio Tirado[c,*]

[a] Departamento de Estadística e Investigación Operativa, Universidad de Valencia, Valencia, Spain
[b] Departamento de Estadística, Matemáticas e Informática, Universidad Miguel Hernández, 03202 Elche, Spain
[c] Departamento de Estadística e Investigación Operativa, Universidad Complutense de Madrid, 28040 Madrid, Spain

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ABSTRACT

This paper approaches a pickup and delivery problem with multiple vehicles in which LIFO conditions are imposed when performing loading and unloading operations and the route durations cannot exceed a given limit. We propose two mixed integer formulations of this problem and a heuristic procedure that uses tabu search in a multi-start framework. The first formulation is a compact one, that is, the number of variables and constraints is polynomial in the number of requests, while the second one contains an exponential number of constraints and is used as the basis of a branch-and-cut algorithm. The performances of the proposed solution methods are evaluated through an extensive computational study using instances of different types that were created by adapting existing benchmark instances. The proposed exact methods are able to optimally solve instances with up to 60 nodes.

1. Introduction

Vehicle routing problems play a very important role in logistics and transportation and have been widely studied during the last decades. Many variants with different characteristics have been considered and a variety of mathematical and computational techniques have been proposed to solve them (see, for example, Toth & Vigo, 2002 and Golden, Raghavan, & Wasil, 2008 for a survey on vehicle routing problems). The Capacitated Vehicle Routing Problem (CVRP) is the reference model for these problems and is defined as follows: given a set of customers, each one demanding a certain amount of a product, and a fleet of vehicles with limited capacity that are based at a given depot, the problem consists of finding a set of minimum cost routes for the vehicles so that all customers are visited exactly once, their demands are fulfilled and the capacity of the vehicles is not exceeded.

The problem approached in this paper is a variant of the so called Vehicle Routing Problem with Pickups and Deliveries (VRPPD) in which the vehicles have to satisfy a set of customer requests where each request specifies the size of the load to be transported, the origin location and the destination location. According to the classification proposed by Berbeglia, Cordeau, Gribkovskaia, and Laporte (2007) this problem belongs to the one-to-one class of pickup and delivery problems.

Note that the CVRP can be considered as a particular case of the VRPPD where all the requests have the depot as the origin location. The VRPPD has been extensively studied in the last three decades (see Berbeglia et al., 2007 and Parragh, Doerner, & Hartl, 2008a, 2008b), incorporating additional constraints that appear in practical situations. In this paper we consider a LIFO (Last-In-First-Out) rule of service, that means only the last pickup customer request that has been loaded can be delivered. Hence, the vehicle servicing an origin location must continue to the corresponding destination or visit another origin location. This condition arises naturally when the vehicles used for transportation are rear-loaded and they have a single access point to their container. The rearrangement of the load when performing deliveries may be possible, but if it is too time consuming, it could be simply forbidden. This is the case, for example, of hazardous materials or very heavy or fragile items. Moreover, we also require that the total duration of each route does not exceed a given limit, where route duration includes traveling times among different locations and service times at each origin/destination for loading and unloading operations. The objective is to find a set of routes with minimum total duration satisfying all the constraints. We refer to this problem as the Multiple Vehicle Pickup and Delivery Problem with LIFO constraints and Maximum Time but it will be abbreviated simply by PDPLT.

To our knowledge, the PDPLT has not been previously studied. However, algorithms have been proposed for related problems. If the capacity and maximum time constraints are relaxed the resulting problem is known as the Traveling Salesman Problem with Pickup,

* Corresponding author. Tel.: +34 913944377; fax: +34 913944606.
E-mail addresses: enrique.benavent@uv.es (E. Benavent), landete@umh.es (M. Landete), Enrique.Mota@uv.es (E. Mota), gregoriotd@mat.ucm.es (G. Tirado).

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Delivery and LIFO constraints (TSPPDL). Carrabs, Cerulli, and Cordeau (2007) develop a branch-and-bound algorithm for this problem and later Cordeau, Iori, Laporte, and Salazar (2010) propose a branch-and-cut algorithm which is able to solve instances with up to 25 requests within a reasonable computing time.

Battarra, Erdogan, Laporte, and Vigo (2010) consider a version of the TSPPDL in which the LIFO rule can be violated, but in this case rehandling costs are incurred. They present models and exact algorithms based on branch-and-cut for this problem, while Erdogan, Battarra, Laporte, and Vigo (2012) have later developed a tabu search metaheuristic.

Cheang, Gao, Lim, Qin, and Zhu (2012) propose several heuristics for a multi-vehicle case where the route length of each vehicle cannot exceed a predetermined limit and the vehicles have unlimited capacity (see also Gao, Lim, Qin, & Zhu, 2011). This problem is called Multiple Pickup and Delivery Traveling Salesman Problem with LIFO and Distance constraints (MTSPPD-LD). Their primary objective is minimizing the number of vehicles used, while the secondary objective is minimizing the total distance traveled. Note that the MTSPPD-LD can be solved as a PDPLT, since minimizing the total distance is equivalent to minimizing the total time, and minimizing the number of vehicles as the primary objective can be achieved by simply adding a big number to the travel times of the arcs leaving the depot.

Another variant that is closely related to the PDPLT was introduced recently by Cherkesly, Desaulniers, and Laporte (2014). In that paper, in addition to imposing LIFO constraints, time windows are associated to every pickup and delivery location. However, they do not consider a maximum time duration for the routes. The authors develop three related branch-price-and-cut algorithms using a three-index formulation of the problem. Their exact solution procedures involve column generation techniques and constrained shortest path subproblems.

The main purpose of this paper is to propose, for the first time, methods to solve the PDPLT. We have developed two mixed integer formulations of the PDPLT and a heuristic procedure that uses tabu search in a multi-start framework. The first formulation is a compact one, that is, the number of variables and constraints grows polynomially with the number of requests, while the second one contains an exponential number of constraints. Separation procedures for several families of the constraints of the non-compact formulation are implemented and embedded in a branch-and-cut algorithm. The compact formulation is easy to use because all the constraints can be included at once, but it is outperformed by the proposed branch-and-cut based on the non-compact formulation for larger instances. The proposed methods are able to solve medium size instances of the PDPLT and provide tight lower and upper bounds for larger instances.

The remainder of the paper is organized as follows. Section 2 introduces the main notation that will be used through the paper. Section 3 presents the compact formulation and Section 4 is devoted to the non-compact formulation, separation algorithms for several families of valid inequalities and the associated branch-and-cut algorithm. Section 5 describes a fast tabu search heuristic for the PDPLT which is embedded into a multi-start framework. The performance of the proposed solution methods is evaluated and analyzed in Section 6, where an extensive computational study is presented. Finally, Section 7 concludes with some final remarks and ideas for future research.

2. Problem notation

The PDPLT is defined on a directed graph $G = (V, A)$.

- $V = P \cup D \cup \{0\}$ is the set of nodes, where $P = \{1, \dots, n\}$ is the set of pickup locations, $D = \{n+1, \dots, 2n\}$ is the set of delivery locations and 0 is the depot. We assume that the delivery location of the load picked up at $i \in P$ is $i + n \in D$.
- A is the set of arcs (i, j) between every pair of nodes $i, j \in V$, $i \neq j$.
- The travel time to traverse each arc (i, j) is denoted by c_{ij}, for all $(i, j) \in A$. It is assumed that travel times satisfy the triangular inequality. Travel times will also be called costs throughout the paper.
- The service time (pickup or delivery) at each node $i \in P \cup D$ is s_i.
- The maximum duration of the route of any vehicle is T.
- For each $i \in P$, $d_i > 0$ is the load of the item that must be picked up at i and delivered at $i + n$. We define, for each $i + n \in D$, $d_{i+n} = -d_i < 0$.
- The number of available vehicles is denoted by m and their capacity by Q.

The goal of the PDPLT is to find a set of routes with minimum total duration that services all the requests, that is, each requested item must be picked up at its origin and delivered at its destination. The duration of the routes include the traversing times among locations and service times and cannot exceed T. The routes must start and end at the depot, the capacity cannot be exceeded, and the LIFO rule must be respected.

3. A compact formulation

In this section we propose a compact formulation for our problem using three families of variables. Binary variables x_{ij} indicate if arc (i, j) is traversed by any vehicle, or not. Continuous variables α_{ij} are typical flow-like variables that in our case indicate the arrival time at node j when the vehicle travels from i to j. Binary variables u_{ik}, defined for any pair of nodes $i \in V$ and $k \in P$, are equal to 1 if the item corresponding to node k is in the vehicle when it leaves node i. These variables are used to prevent subtours, to express the capacity constraints and to assure that the pickup and the delivery nodes associated to a given request belong to the same route. This last condition is not easy to impose because variables x_{ij} do not allow to identify which vehicle is traveling from i to j.

Variables u_{ik} and their corresponding constraints are inspired by the Miller–Tucker–Zemlin formulation for the Travelling Salesman Problem (see Miller, Tucker, & Zemlin, 1960) that were improved later and used for the Vehicle Routing Problem by Kara, Laporte, and Bektas (2004). Let us define coefficients β_{ik} as follows: $\beta_{kk} = 1$ and $\beta_{k+n,k} = -1$ for all $k \in P$, and $\beta_{ik} = 0$ for any $i \in V, i \neq k, n+k$. Note that coefficients β_{ik} indicate that item k can only be picked up in node k ($\beta_{kk} = 1$) and delivered in node $k+n$ ($\beta_{k+n,k} = -1$). For instance, if $n = 3$, it is

$$\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Proposition 1. *Constraints (1) are valid for the PDPLT.*

$$u_{ik} - u_{jk} + x_{ij} + (1 - \beta_{ik} - \beta_{jk})x_{ji} \leq 1 - \beta_{jk} \quad \forall (i, j) \in A, \ k \in P \qquad (1)$$

Proof. Let us first prove the validity of constraints (2).

$$u_{ik} - u_{jk} + x_{ij} \leq 1 - \beta_{jk} \quad \forall (i, j) \in A, \ k \in P \qquad (2)$$

Note that the definition of variables u_{ik} implies that $u_{kk} = 1$ and $u_{n+k,k} = 0$ for all $k \in P$. If $x_{ij} = 0$, then the possible values of $u_{ik} - u_{jk}$ are $-1, 0, 1$ and, since the RHS is always greater than or equal to zero, we only have to analyze the case where $u_{jk} = 0$ and $u_{ik} = 1$, but then $\beta_{jk} \neq 1$ and the constraint holds. If $x_{ij} = 1$, then $u_{ik} - u_{jk} \leq -\beta_{jk}$. If $u_{ik} = 1$, then either $u_{jk} = 1$ and $\beta_{jk} = 0$ or $u_{jk} = 0$ and $\beta_{jk} = -1$ ($j = k + n$). The case where $x_{ij} = 1$ and $u_{ik} = 0$ is proved similarly.

These constraints can be improved by introducing an extra term, in a similar way as in Kara et al. (2004) : $u_{ik} - u_{jk} + x_{ij} + \gamma_{ji}x_{ji} \leq 1 - \beta_{jk}$. It can be observed that the largest value that can be given to γ_{ji} is $(1 - \beta_{ik} - \beta_{jk}) \geq 0$. □