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Invited Review

Mathematical programming techniques in water network optimization

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ABSTRACT

In this article we survey mathematical programming approaches to problems in the field of drinking water distribution network optimization. Among the predominant topics treated in the literature, we focus on two different, but related problem classes. One can be described by the notion of network design, while the other is more aptly termed by network operation. The basic underlying model in both cases is a nonlinear network flow model, and we give an overview on the more specific modeling aspects in each case. The overall mathematical model is a Mixed Integer Nonlinear Program having a common structure with respect to how water dynamics in pipes are described. Finally, we survey the algorithmic approaches to solve the proposed problems and we discuss computation on various types of water networks.

1. Introduction

In classic network flow problems the task is to route a flow through a network from a set of sources to a set of sinks. This point of view can become coarse when dealing with pressurized water networks, where the fluid is transported in pipes with no air contact and thus possibly varying pressure levels. A first step toward accurately modeling the physical aspects of such networks, is the introduction of pressure variables at nodes in addition to flow variables on arcs. In this modeling enhancement, what actually induces a flow between two nodes is explained by a pressure difference. To subsume a broader field of applications, the additional variables in such approaches are also referred to as node potentials, including as well the electric potential of a point in an electric circuit. Pressure again is an important quantity in gas networks. In drinking water distribution network optimization, such a modeling approach has experienced eminent interest in order to develop physically sound models for real-world applications. The drawback of the resulting accuracy gain is the fact that the relation between flow and potential difference usually leads to nonlinear equations. Together with discrete decisions that can be made regarding different network elements, this puts the optimization tasks faced here in the context of Mixed Integer Nonlinear Programming (MINLP).

In the following we focus on surveying topics related to the optimization of drinking water distribution networks. We will drop the attributes *drinking* and *distribution* and for this text establish the convention that the term water networks subsumes anything that is named by drinking water distribution networks, water supply systems, or combinations of the two. Other types of water networks, such as waste water networks (Rauch & Harremoës, 1999) or water usage and treatment networks in chemical plants (Huang, Chang & Ling, 1999) are not considered here. Moreover, we point out that besides design and operation there are other topics related to the optimization of water networks, for example the containment detection problem (Lorenz, Laird, Biegler & Van Bloemen Waanders, 2006) or topics related to water quality management (Rossman & Boulos, 1996), which are not covered here. The term network underlines the fact that we deal with applications in which the underlying structure can be modeled by a graph in a mathematical sense. Out of the different stages into which water network optimization can be subdivided, we focus on the somewhat different tasks of *optimal design of water networks* on the one hand and *optimal operation of water networks* on the other. On both sides, one assumes to have an underlying network with a fixed topology, i.e., a fixed set of nodes and arcs representing sources, sinks, pipes, pumps, valves, and tanks. The former of the two optimization tasks, i.e., the design problem, usually disregards pumps, valves and tanks. One then seeks to choose for each pipe in the network a diameter among a discrete set of commercially available diameters in a cost-minimal way, while maintaining the satisfiability of all costumer demands located at sinks. The diameter has an important impact not only on the pipe's capacity, but also on the pressure distribution in the network. The operation problem instead typically assumes fixed pipe diameters but allows for the modeling of pumps, valves, and tanks. The task is then to operate pumps and valves, which again affect flow

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and pressure distribution, over a certain time horizon in order to satisfy the customer demands, while minimizing the operational costs mainly arising from power consumption of pumps. In its full-scale form, the operation problem hence incorporates the aspect of time into the model and is thus a *dynamic* problem. Conversely, in order to model the design problem, no time parameter is neither needed nor generally used in the literature, which is why we consider it a *static* problem.

The fact that active elements (pumps, valves, and tanks) are disregarded in design problems, and that there are only passive elements (pipes), is prevalent in the literature, although this does not necessarily have to be so. However, unifying design and operational phases in a single model bears some difficulties, and we will address this issue shortly in Section 5.3.

In spite of these differences, there are still some obvious similarities from a mathematical point of view, due to the way water dynamics in a pipe is described. Typically, the majority of the arcs in a network is constituted by pipes, and the equation associated with a pipe will be at the heart of the present survey. As mentioned earlier, the problems we consider here belong to the class of MINLPs and, in their general form, they involve two sources of non-convexity to model the flow of water into pipes (depending on the pressure difference at the nodes) and to deal with discrete choices. Thus, both water network design and operation are NP-hard problems, and in the present paper we are interested in mathematical programming approaches, i.e., methods that explicitly use a mathematical programming model. Those methods exploit (different variants of) different algorithmic paradigms to solve MINLPs, including Mixed Integer Linear Programming (MILP) techniques. We try to provide an overview of how different techniques succeed in different situations.

In the last decade, MINLP approaches have experienced increasing popularity, whereas before that, optimization problems related to water networks were prevalently attacked by (meta-)heuristic methods, see, e.g., da Conceição Cunha and Ribeiro (2004) and De Corte and Sörensen (2012), without explicitly using a mathematical programming formulation. In the following, we do not dwell on the variety of those heuristic approaches that exist in the field. For a broader discussion of topics related to water networks that covers also aspects that are not of algorithmic nature, we refer the reader to Coelho and Andrade-Campos (2014). Finally, it is worth mentioning the existence of Partial Differential Equation (PDE) approaches in the field of water network optimization, see, e.g. Laird, Biegler and Van Bloemen Waanders (2007), reflecting the fluid-dynamic nature of the topic. Again, this is outside the scope of the present survey.

2. Modeling

To begin with, we present the main modeling aspects found in the literature concerning the design and the operation problems. Regarding the various network elements, different variants at different levels of detail can be found. The most detailed modeling description of the relevant aspects is provided in Burgschweiger, Gnädig and Steinbach (2009b). A network is naturally represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where nodes stand for sources and sinks and arcs stand for pipes, pumps, and valves. Tanks are usually modeled as nodes, but this is not always true, see, e.g., Morsi, Geißler and Martin (2012).

2.1. Flow and pressure

A main difference between the design and the operation problems is of course the contrast between the static and the dynamic setting. In the latter, in principle all variables and parameters can be made continuous-time dependent on $t \in [0, T]$, where $[0, T] \subset \mathbb{R}$ is the considered time horizon. However, to get a tractable optimization model, time is usually discretized and the quantities depend on the discrete time period $n \in \{1, \dots, N\}$ of length $\tau \in \{\tau_1, \dots, \tau_N\}$. A typical planning horizon is one day, divided in 24 hourly periods. In Burgschweiger, Gnädig and Steinbach (2009b) it is pointed out that the discretization has another practical motivation. Namely, demand forecasts and electricity price tariffs are usually given for discrete and not continuous time. In the following we will highlight the time dependency of variables or parameters with a superscript n only when different periods are involved in an equation or constraint. Otherwise, the equation usually has to be imposed in every time period. In a static setting there is of course no time dependency to be highlighted. In any case, we introduce a flow variable q_a on each arc $a \in \mathcal{A}$. By allowing the flow to take negative values, a directed graph accounts for its both possible directions: a positive flow q_a on an arc $a = (i, j)$ means that it goes from i to j, while a negative value of q_a stands for a flow of amount $|q_a|$ from j to i. It is as well possible to allow only positive flow values and account for the directions with a binary variable.

From classical network flow problems one inherits the flow conservation constraints. For a node $i \in \mathcal{N}$ with demand d_i, which for the moment is assumed to be a constant, and the set of incoming and outgoing arcs, δ_i^- and δ_i^+, respectively, one has the linear constraint

$$\sum_{a \in \delta_i^-} q_a - \sum_{a \in \delta_i^+} q_a = d_i. \tag{1}$$

Sinks have a positive demand, which means that water actually leaves the network at those nodes. At sources, often called reservoirs in the water context, constraint (1) is usually not imposed. Otherwise, one can model d_i as a variable that can assume only non-positive values, possibly bounded from below. Of course, there can be nodes with zero demands. Sometimes all nodes with positive or zero demand are called junctions. In real-world applications, there is usually uncertainty in the data, for example in the demand d_i. This aspect has been addressed in (meta)heuristic approaches, see, e.g., Babayan, Kapelan, Savic and Walters (2005). However, in the Mathematical Programming literature, which is focus of the present paper, it has been very rarely taken into account. Thus, in this survey we consider approaches that assume deterministic data for the demands, which is, in any case, quite reasonable because generally one wants to establish a network design or operation that is feasible also in worst-case scenarios.

Next, one introduces the node potential variables h_i, $i \in \mathcal{N}$, representing the hydraulic head of the water at a node. This variable represents the pressure: In fluid dynamics, total head is the total energy per unit weight of fluid and is the sum of potential energy (elevation), pressure energy due to the pressure exerted on its container (pressure) and kinetic energy (velocity head). Consistent with this definition, total head and pressure are expressed dimensionally as a length. Due to the small value of velocity head in relation to the other two terms of the sum, the velocity head is generally neglected and the total head is assimilated to the hydraulic head, given by the sum of elevation and pressure.

A set of constraints that is regularly found in diverse optimization approaches arises from the fact that the two groups of variables introduced above are typically bounded. The absolute value of the flow is bounded from above due to the capacity of the arcs. For example, taking into account the maximum flow velocity that is allowed in a pipe a, $v_a^{\max}$, the flow bound can be written (Bragalli, D'Ambrosio, Lee, Lodi & Toth, 2012) as

$$-\frac{\pi}{4} v_a^{\max} D_a^2 \le q_a \le \frac{\pi}{4} v_a^{\max} D_a^2, \tag{2}$$

where D_a is the diameter of pipe a. The node potentials have to stay between certain bounds in order to guarantee minimum and maximum pressure levels at the nodes. Usually, the node potentials are fixed at source nodes, reflecting the fact that at sources water is not pressurized, but it exploits a fixed geographical height. We will see in Section 6 that the bounds on the potential values often constitute the physical bottleneck in a water network.