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A variable neighborhood search for the capacitated vehicle routing problem with two-dimensional loading constraints

Lijun Wei [a], Zhenzhen Zhang [b,*], Defu Zhang [c], Andrew Lim [d,e]

[a] School of Information Technology, Jiangxi University of Finance and Economics, Nanchang 330013, Jiangxi, People's Republic of China
[b] Department of Management Sciences, City University of Hong Kong, Tat Chee Ave, Kowloon Tong, Hong Kong
[c] Department of Computer Science, Xiamen University, Xiamen 361005, China
[d] School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371, Singapore
[e] International Center of Management Science and Engineering, School of Management and Engineering, Nanjing University, Nanjing 210093, People's Republic of China

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ABSTRACT

This paper addresses the capacitated vehicle routing problem with two-dimensional loading constraints (2L-CVRP), which is a generalized capacitated vehicle routing problem in which customer demand is a set of two-dimensional, rectangular, weighted items. The objective is to design the route set of minimum cost for a homogenous fleet of vehicles, starting and terminating at a central depot, to serve all the customers. All the items packed in one vehicle must satisfy the two-dimensional orthogonal packing constraints. A variable neighborhood search is proposed to address the routing aspect, and a skyline heuristic is adapted to examine the loading constraints. To speed up the search process, an efficient data structure (Trie) is utilized to record the loading feasibility information of routes, but also to control the computational effort of the skyline spending on the same route. The effectiveness of our approach is verified through experiments on widely used benchmark instances involving two distinct versions of loading constraints (*unrestricted* and *sequential* versions). Numerical experiments show that the proposed method outperforms all existing methods and improves or matches the majority of best known solutions for both problem versions.

1. Introduction

This paper considers an important extension of the classical vehicle routing problem (VRP), called capacitated vehicle routing problem with two-dimensional loading constraints (2L-CVRP). This problem is a combination of two most important NP-hard optimization problems in distribution logistics, the capacitated vehicle routing problem (CVRP) and two-dimensional bin packing problem (2BPP). In the 2L-CVRP, the demand of customers is formed by several two-dimensional rectangular weighted items, while vehicles are identical with the same weight capacity and loading surface. The purpose is to design routes with minimum cost for given vehicles to serve all the customers and, at the same time, load the corresponding items into the vehicles successfully.

The 2L-CVRP is particularly important in both practical and theoretical aspects. The CVRP is a simplified model for practical operators. However, in the real world, logistics managers usually have to deal with routing and packing problems simultaneously. Moreover, because of the weight, fragility or the large height of freight, numerous real life applications involve the delivery of rectangular-shaped items that cannot be stacked on each other, such as household appliances, delicate pieces of furniture, etc. Therefore, 2L-CVRP has an obvious commercial value. From a theoretical perspective, 2L-CVRP is composed of two NP-hard problems (CVRP and 2BPP); thus, it is also a challenging NP-hard problem of high complexity. The CVRP (Toth & Vigo, 2002) has been extensively studied (Gendreau, Hertz, & Laporte, 1994; Prins, 2004). The loading component of 2L-CVRP is closely related to the two-dimensional bin packing problem (2BPP), which aims at packing a given set of rectangular items into the minimum number of identical rectangular bins. For more related literature, the reader is referred to Lodi, Martello, and Vigo (1999) and Lodi, Martello, and Monaci (2002).

Routing and packing problems have been studied intensively but separately, with the combined problem only being introduced in recent years. The 2L-CVRP was first presented by Iori, Salazar-González, and Vigo (2007), and only small-scale instances were solved via an exact algorithm based on the branch-and-cut technique. For larger scale

* Corresponding author. Tel.: +852 64067667/54323245; fax: +852 34420188.
E-mail addresses: villagerwei@gmail.com (L. Wei), zhenzhenzhang222@gmail.com (Z. Zhang), dfzhang@xmu.edu.cn (D. Zhang), alim.china@gmail.com (A. Lim).

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problems, Gendreau, Iori, Laporte, and Martello (2008) proposed the first meta-heuristic approach, Tabu Search (TS). Then, Zachariadis, Tarantilis, and Kiranoudis (2009) developed the Guided Tabu Search (GTS), which incorporates the rationale of Tabu Search and Guided Local Search, and relies on a bundle of heuristics to check the feasibility of loading. The Extended Guided Tabu Search (EGTS) and simulation annealing (SA), which introduce a new scoring-based heuristic to improve packing, were proposed by Leung, Zhou, Zhang, and Zheng (2011) and Leung, Zheng, Zhang, and Zhou (2010), respectively. For a population-based algorithm, the effective saving-based Ant Colony Optimization (ACO) was presented by Fuellerer, Doerner, Hartl, and Iori (2009), and its performance was proven to be quite satisfactory. Recently, Duhamel, Lacomme, Quilliot, and Toussaint (2011) proposed the Greedy Randomized Adaptive Search Procedure combined with Evolutionary Local Search (GRASP × ELS) algorithm, whereby the loading component is examined via solving the resource constrained project scheduling problem (RCPSP). This algorithm outperforms all previous methods and obtains new better solutions for several instances; however, only the *Unrestricted* version of 2L-CVRP was solved. Lately, Zachariadis, Tarantilis, and Kiranoudis (2013) proposed an innovative compact meta-heuristic, named as promise routing-memory packing (PRMP), which obtains excellent performance and improves best known solutions for many instances.

The more general problem, capacitated vehicle routing problem with three-dimensional loading constraints (3L-CVRP), has received increasing attention from researchers. Several algorithms proposed for solving 2L-CVRP were extended to 3L-CVRP, e.g., TS (Gendreau, Iori, Laporte, & Martello, 2006), GTS (Tarantilis, Zachariadis, & Kiranoudis, 2009), ACO (Fuellerer, Doerner, Hartl, & Iori, 2010), and GRASP × ELS (Lacomme, Toussaint, & Duhamel, 2013). In addition, Ma, Zhu, and Xu (2011) provided a combined tabu search with local search. Bortfeldt (2012) introduced a hybrid algorithm, which employs tabu search for routing and tree search algorithm for loading. Zhu, Qin, Lim, and Wang (2012) developed a two-stage tabu search for routing, adopted enhanced heuristics for loading and obtained excellent results. Iori and Martello (2010) provided a review in regard to vehicle routing problems with two- and three-dimensional loading constraints. Wei, Zhang, and Lim (2014) introduced a heterogeneous fleet vehicle routing problem with three-dimensional loading constraints (3L-HFVRP).

The VNS (Hansen, Mladenović, Brimberg, & Pérez, 2010) has been proven quite effective to solve the VRP and its variants (Fleszar, Osman, & Hindi, 2009; Hemmelmayr, Doerner, & Hartl, 2009; Imran, Salhi, & Wassan, 2009; Paraskevopoulos, Repoussis, Tarantilis, Ioannou, & Prastacos, 2007). Motivated by these successes, this paper develops a variable neighborhood search (VNS) for 2L-CVRP, which incorporates a skyline heuristic for solving the loading component. To the best of our knowledge, this is the first implementation of VNS for 2L-CVRP. In our method, an insertion-based heuristic is adopted to generate the initial solution, which is used for further improvement. Regarding the routing aspect, VNS can explore the solution space systematically and effectively. Moreover, in order to ensure the loading feasibility of routes, an effective skyline heuristic is employed to solve the packing part of the problem. To accelerate the procedure, a data structure (Trie) is used to record the loading feasibility information, which avoids checking the visited routes repeatedly. In addition, it is also used to control the number of times the skyline heuristic is called for the same route. The proposed algorithm was tested extensively on 2L-CVRP benchmark instances and compared with previously published algorithms. The results demonstrate that our algorithm outperforms all other methods and identifies many new best solutions, especially for large scale instances.

The remainder of this paper is organized as follows. Section 2 describes the problem in detail. Section 3 presents the proposed variable neighborhood search methodology, while Section 4 provides a detailed description of the skyline heuristic for examining the loading subproblem. The extensive computational results on benchmark instances and comparisons with other algorithms are presented in Section 5. Finally, the conclusions are given in Section 6.

2. Problem description

The 2L-CVRP is defined on a complete undirected graph $G = (V, E)$, where $V = \{0, 1, \dots, N\}$ is the vertex set and $E = \{(i, j) | i, j \in V, i \neq j\}$ is the edge set. Vertex 0 represents the central depot and the set of vertices $\{1, 2, \dots, N\}$ denotes the location of customers. Each edge $(i, j) \in E$ associates with a travel cost c_{ij} that corresponds to the cost for going from vertex i to j or from vertex j to i. At the central depot, a fleet of K homogeneous vehicles is available. Each vehicle has weight capacity D and a two-dimensional rectangular loading surface of width W and length L. The loading area is denoted as $A = W \times L$. The demand of each customer i $(i = 1, \dots, N)$ is defined as a set of m_i rectangular items denoted as I_i, and the total weight of I_i is d_i. Each item $I_{ir} \in I_i (r = 1, \dots, m_i)$ is characterized by a specific width w_{ir} and length l_{ir}. In addition, the total area of items I_i is denoted as $a_i = \sum_{r=1}^{m_i} w_{ir} l_{ir}$. For 2L-CVRP, a feasible solution must satisfy the following constraints:

(a) Every vehicle starts and terminates its route at the central depot.
(b) All the customers must be served by using no more than the given K vehicles.
(c) Each customer is visited exactly once, and all the demanded items must be loaded into the vehicle.
(d) The capacity, length, and width of every vehicle cannot be exceeded by loaded items.
(e) Each item has a fixed orientation that cannot be rotated. In other words, each item is loaded with its width (length) parallel to the corresponding width (length) of the vehicle surface.
(f) All items of customers assigned to the same route must be loaded into the vehicle successfully without any overlap.

The objective of 2L-CVRP is to minimize the total travel cost of routes which serve all the customers and satisfy all the constraints, as well. This paper considers two versions of this problem: *Unrestricted* 2L-CVRP and *Sequential* 2L-CVRP. The *Unrestricted* 2L-CVRP is described in the previous paragraph, which only concerns loading items into the vehicle. *Sequential* 2L-CVRP considers both loading and unloading operators, in which an additional *LIFO (last in first out) constraint* is imposed: when visiting a customer, his or her items can be unloaded by straight movements parallel to the length dimension of the vehicle surface, without the need to rearrange items that belong to other customers in the same route. In other words, no item of customer j in the same route to be served later than customer i can be placed between items of i and the rear door of the vehicle. This arises in practical cases when it is difficult to move the items due to their weight or fragility, especially when the items are unloaded by means of forklift trucks from the rear door of vehicles. Fig. 1 gives an example of the two versions.

3. The VNS algorithm for 2L-CVRP

Variable Neighborhood Search (VNS) is first proposed by Mladenović and Hansen (1997) to solve combinatorial and global optimization problems. VNS is derived from the idea of systematically changing neighborhoods during the search. The underlying theory to obtain a better local optimum is that a local optimum under one neighborhood structure is not necessary so far from another.

In the classical implementation of VNS algorithm, four key components should be specified: (i) method to construct an initial solution;