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Start-up demonstration tests with sparse connection

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ABSTRACT

Based on the concept of sparse connection, three start-up demonstration tests with sparse connection are introduced which are called CSTF with sparse d_1, TSCF with sparse d_2 and CSCF with sparse d_3 and d_4. The traditional start-up demonstration tests such as CSTF, TSCF and CSCF are special cases of these new tests. Furthermore, the new tests exhibit obvious improvement in test efficiency. In this paper, by using the finite Markov chain imbedding approach, several probabilistic indexes are given for these new start-up demonstration tests based on the assumption that the tests are i.i.d. case. The analyses are also extended to independent and non-identical and Markov dependent cases. In addition, procedures are provided in order to determine the optimal parameters needed in a demonstration test for selecting the products to meet the reliability requirement. Three comparison analyses are finally presented in order to illustrate the high efficiency of these new start-up demonstration tests and the effectiveness of this method.

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Abbreviation

Acronym

CS	consecutive successes
CSTF	consecutive successes total failures
TSCF	total successes consecutive failures
TSTF	total successes total failures
CSCF	consecutive successes consecutive failures
CSDF	consecutive successes distant failures
TSCSTF	total successes consecutive successes total failures
TSCSTFCF	total successes consecutive successes total failures consecutive failures
R_1-CS/TS/R_2-CF/TF R_1	runs of consecutive successes total successes R_2 runs of consecutive failures total failures
CSTF with sparse d_1	consecutive successes with sparse d_1 total failures
TSCF with sparse d_2	total successes consecutive failures with sparse d_2
CSCF with sparse d_3 and d_4	consecutive successes with sparse d_3 consecutive failures with sparse d_4

Notation

k_C	the number of consecutive successes required for acceptance
k_T	the number of total successes required for acceptance
f_C	the number of consecutive failures required for rejection
f_T	the number of total failures required for rejection
d_i	the maximum number of failed start-ups between two successful start-ups or the maximum number of successful start-ups between two failed start-ups
L	the test length until termination of the test
$E(L)$	expected value of L
p, q	probability of success, failure at each trial (i.i.d.)
α	the producers' risk
β	the consumers' risk

1. Introduction

Start-up demonstration tests are very important to spacecraft propulsion systems, emergency lighting systems, and other various engineering systems, because if these systems fail to start up, it will result in serious consequences such as large economic losses and casualties. Hence, a start-up demonstration test is very important as a method for selecting equipment with high start-up reliability during adoption process. By this method, a customer should judge whether the start-up reliability of the equipment meets the reliability requirement before purchasing it by observing the outcomes of a set of tests on the equipment.

In a start-up demonstration test, the judging criterion of start-up reliability levels is related to the numbers of successes and failures in several trials which form the acceptance and rejection criteria.

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In general, the acceptance criterion can be divided into two criteria: consecutive successes and total successes. The rejection criterion can also be fallen into two categories: consecutive failures and total failures. Based on the combinations of different acceptance and rejection criteria, start-up demonstration tests can be divided into several categories such as CS, CSTF, TSCF, CSCF, TSTF, CSDF, TSCSTFCF, etc. The simplest and original start-up demonstration test is called CS (consecutive successes) which has been presented by Hahn and Gage (1983), and Viveros and Balakrishnan (1993). By adding a rejection criterion to CS models, Balakrishnan and Chan (2000) proposed a new start-up demonstration test called CSTF (consecutive successes total failures). In terms of the assumption that the start-ups are i.i.d. cases, they also derived the probability generating function of the length of the start-up demonstration test. After that, Smith and Griffith (2005, 2008) easily obtained the probabilities of acceptance and rejection, the probability mass function, distribution function, mean and variance of the test length by using the Markov chain approach which is also adapted to non i.i.d. cases. They also studied procedures to find the appropriate parameters of start-up demonstration tests and methods to estimate the start-up reliability of the equipment. Besides, they analyzed and compared the advantages of different tests in certain situations by introducing two new start-up demonstration tests: TSTF (total successes total failures) and CSCF (consecutive successes consecutive failures). By conditioning on the time of the first failure, Martin (2004) derived several results for demonstration tests of the start-up reliability of equipment. He also gave recursive formulas for computing the probability distribution of the number of start-ups and the probability of acceptance or rejection of the equipment in a specified number of trials. The auxiliary Markov chains were used by Martin (2008), who derived probabilistic results for five types of start-up demonstration tests, with start-ups that are Markovian of a general order. Antzoulakos, Koutras, and Rakitzis (2009) introduced the CSDF (consecutive successes distant failures) model and studied a new acceptance/rejection rule which is suitable to use in start-up demonstration tests. As extended models of CSTF, TSCSTF (total successes consecutive successes total failures) and R_1-CS/TS/R_2-CF/TF (R_1 runs of consecutive successes, total successes, R_2 runs of consecutive failures, total failures) were proposed by Gera (2010) and Zhao (2014) respectively. Gera (2011) also improved the CSTF test to TSCSTFCF test (total successes consecutive successes total failures consecutive failures) and then he generalized this model to include dependent tests according to the previous-sum dependent model (Gera, 2013). As a generalization of CSDF model, Gera (2013) introduced TSCSTFDF test which has significant reduction in the expected number of required tests together with improved second-order statistics (standard deviation). Zhao (2013) extended the application of start-up demonstration tests to the case of products with start-up delay, and he also presented the method for evaluating the indexes related to the test length by using the finite Markov chain imbedding approach. Yalcin and Eryilmaz (2012) studied the TSTF test under previous-sum dependent model and derived the main characteristics for the TSTF test in such condition.

The naming of a start-up demonstration test model is very straightforward, for example, if k_C consecutive successes are observed prior to a total of f_T failures, then the equipment will be accepted; inversely, if f_T total failures are observed prior to k_C consecutive successes, it will be rejected (Smith & Griffith, 2005). We call this CSTF test. The start-up demonstration test will be terminated if the equipment is either accepted or rejected. The definitions of other start-up demonstration tests are similar to that of CSTF test only by replacing "consecutive" and "total" appropriately.

In this paper, the previous start-up demonstration tests are further generalized by considering the concept of sparse connection, and three new start-up demonstration tests are proposed which are called CSTF with sparse d_1, TSCF with sparse d_2 and CSCF with sparse d_3 and d_4. In Section 2, the models of different start-up demonstration tests with sparse connection are presented. In Section 3, by using the finite Markov chain imbedding approach, it is possible to obtain the probability mass function, the distribution function, the mean of the test length, the acceptance and rejection probabilities of these three new models. Besides, the analyses are generalized to non i.i.d. cases. Procedures to find the optimal parameter values of start-up demonstration tests with sparse connection are proposed in Section 4. Section 5 presents two comparison analyses between new and traditional models with numerical examples showing that the new models are always more efficient than the traditional models. The summary and some concluding remarks are given in Section 6.

2. Modeling of start-up demonstration tests with sparse connection

Based on the CSTF, TSCF, CSCF and the concept of sparse connection proposed by Zhao, Cui, and Kuo (2007), we present three start-up demonstration tests with sparse connection: CSTF with sparse d_1, TSCF with sparse d_2 and CSCF with sparse d_3 and d_4. As to the start-up demonstration tests with sparse connection in this paper, we only consider the cases of that there are two states in each single trial, success and failure. The difference between these new start-up demonstration tests and ordinary start-up demonstration tests is the redefinition of "consecutive". The concept of sparse connection is introduced as follows.

Definition 1. The start-up results are outputted in turn, forming a sequence of outcomes. If there are no successful start-ups between two adjacent successful start-ups and the number of failed start-ups between the two successful start-ups is d_i or less, then the two successful start-ups are called two consecutive successes with sparsed_i. Likewise, if there are no failed start-ups between two adjacent failed start-ups and the number of successful start-ups between them is d_i or less, these are two consecutive failures with sparse d_i.

For example, an outcome sequence of twenty trials is {*SFFFSSSFSFSFFSFFFFSS*}, based on Definition 1, consecutive successes or failures with different d_i are shown in Fig. 1.

Definition 2. In a start-up demonstration test, the equipment under test will be accepted if k_C consecutive successes with sparse d_1 are observed prior to a total of f_T failures; it will be rejected, if f_T total failures are observed prior to k_C consecutive successes with sparse d_1. We call this model CSTF with sparse d_1.

Analogously, there is also a start-up demonstration test called TSCF with sparse d_2. The difference between TSCF with sparse d_2 and CSTF with sparse d_1 lies in sparse connection and the value of d_i. The former is consecutive failures with sparse d_2 and the latter is consecutive successes with sparse d_1. The acceptance and rejection criteria of TSCF with sparse d_2 are similar to that of CSTF with sparse d_1 only with interchanging the "successes" with "failures" properly.

Definition 3. In a start-up demonstration test, the test will be terminated either k_C consecutive successes with sparse d_3 are observed prior to f_C consecutive failures with sparse d_4, leading to acceptance of

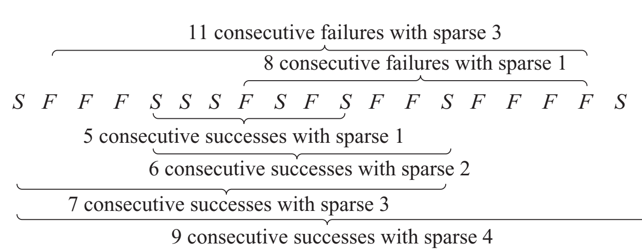

Fig. 1. Consecutive successes and failures with different d_i.

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