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Cost analysis for multi-component system with failure interaction under renewing free-replacement warranty

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ABSTRACT

In a multi-component system, the assumption of failure independence among components is seldom valid, especially for those complex systems with complicated failure mechanism. For such systems, warranty cost is subject to all the factors including system configuration, quality of each component and the extent of failure dependence among components. In this paper, a model is developed based on renewing free-replacement warranty by considering failure interaction among components. It is assumed that whenever a component (subsystem) fails, it can induce a failure of one or more of the remaining components (subsystems). Cost models for series and parallel system configurations are presented, followed by numerical examples with sensitivity analysis. The results show that, compared with series systems, warranty cost for parallel systems is more sensitive to failure interaction.

1. Introduction

A warranty is an assurance by the manufacturer (vendors, sellers, or third parties) to the buyer which requires manufacturers to offer pre-defined compensation to buyers if the product or service fails to meet the standards under normal usage within warranty duration (Vahdani, Mahlooji, & Eshraghnia Jahromi, 2013; van der Heijden & Iskandar, 2013; Xie, Liao, & Zhu, 2014). Different types of warranties are offered by manufacturers based on the characteristics of products, e.g., the products' complexity, reparability, and reliability (Lo & Yu, 2013). These characteristics are closely related to the key target of warranty research – warranty cost modeling.

There are numerous warranty policies adopted in cost modeling. Detailed studies can be found in Sheu and Chien (2005), Bouguerra, Chelbi, and Rezg (2012), Park, Mun Jung, and Park (2013), Shafiee and Chukova (2013), Xie and Liao (2013). Specifically, cost modeling under renewing free-replacement policy (RFRW) is extensively studied in literature (Chien, 2008, 2012; Wu & Xie, 2008; Darghouth, Chelbi, & Ait-Kadi, 2012; Vahdani, Chukova, & Mahlooji, 2011; Yeh, Chen, & Chen, 2005). Under the RFRW policy, a product failed within the warranty period is replaced by a new one with a full warranty. Bai and Pham (2006) proposed a renewing 'full-service' warranty for multi-component system, which assumes an extra perfect maintenance upon system failure after the normal free replacement under RFRW.

Related research has been focused increasingly on the warranty studies for complex systems. The main reason is due to the need for an accurate estimation of warranty cost for those systems as the warranty cost could be very high. When a complex system is treated as a 'single-component' system or a 'black box', the inner structure information is ignored (Wu and Xie, 2008; Ye, Murthy, Xie, & Tang, 2013). It is desirable to consider a different warranty policy for multi-component system when failure of several units instead of one unit results in the warranty cost (Bai & Pham, 2006; Scarf & Majid, 2011). When shifting the reliability analysis from single item to multiple items, failure dependence is a common phenomenon that cannot be neglected (see Peng, Coit, & Feng, 2012; Tsoukalas & Agrafiotis, 2013; Yu, Chu, Châtelet, & Yalaoui, 2007). Murthy and Nguyen (1985) formulated three different types (Types I–III) of failure interaction for a two-component system. Type I failure interaction assumes that whenever a component fails, it can induce a simultaneous failure of one or more of the remaining components of the system. They define this simultaneous failure of the remaining components as 'induced failure', compared with 'natural failure' described by components' lifetime distribution without failure interaction. Type II failure interaction is known as failure rate interaction, which assumes a change on failure rate of components whenever a component fails. It is further discussed by Zequeira and Bérenguer (2005), Lai (2008), and Golmakani and Moakedi (2012). A combination model of Type I and Type II is the Type III failure interaction (Murthy & Nguyen, 1985). So far, failure interaction is mainly discussed in maintenance models. Warranty cost models under failure interaction are seldom explored.

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In this paper, we present a warranty cost model with Type I failure interaction under a type of renewing free-replacement warranty (RFRW). The target system is composed of multiple components and is repairable. Compared with previous works, the following two important extensions are made in this paper. (1) Instead of assuming independent failures among components, we derive the expected warranty cost based on certain failure interaction model. (2) We consider failure interaction between each two components instead of the failure interaction between one component and the whole remaining components. Warranty cost models for system configurations such as series and parallel are discussed separately, which provides a basis for future study of even more complex system configurations, such as parallel-series, series-parallel, hierarchical, and k-out-of-n. Since failure dependence is a common problem in complex system which affects both system reliability and service cost, this paper can help decision-makers better evaluate system reliability and reduce risk in estimating future warranty cost.

The rest of this paper is organized as follows. Section 2 introduces the model and the assumptions for multi-component system subject to failure interaction, maintenance strategy and warranty policy. Sections 3 and 4 formulate the specific warranty cost models under series and parallel structure. Section 5 gives numerical examples for a 3-component parallel system; comparison with series system is made to illustrate the result. Conclusions and potential extensions are made in Section 6.

2. Assumptions and specifications

This section provides preliminary assumptions and model specifications in RFRW and failure interaction.

2.1. Assumptions of renewing free-replacement warranty (RFRW)

We follow the standard assumptions on RFRW such as those presented in Bai and Pham (2006). A pre-specified warranty period is denoted by w. $T_S = \sum^{t_i} + w$ denotes the total time under warranty (warranty cycle), and $t_i (t_i < w)$ is the system lifetime.

Several assumptions on RFRW for a multi-component system are made as follows:

1. Failed components within T_S are fully replaced. Meanwhile, warranty terms are renewed by manufacturers.
2. When the system fails, a perfect maintenance effort (Borrero & Akhavan-Tabatabaei, 2013; Chen, Ye, & Xie, 2013; Shafiee, Finkelstein, & Zuo, 2013) is conducted for the survived components to reduce the chances of future system failures. Upon this assumption, the system is restored to the state of "as good as new" after each warranty service due to system failure.
3. Warranty cost is subject to two items: one is the replacement cost of the failed components; the other is the maintenance cost due to system failure.
4. In order to reduce the complexity of the cost model, maintenance cost per system failure is assumed to be a constant (Liu, Xu, Xie, & Kuo, 2014; Wang, 2002). It is defined as the average maintenance service cost throughout all the system failures within warranty cycle T_S.

2.2. Failure interaction

We consider Type I failure interaction among components, which can be interpreted as follows (Murthy & Nguyen, 1985):

1. Let $\Omega = \{1, 2, \ldots,\}$ denote the number of components. Consider an n-component system with series or parallel structure. For each component, there are two types of failures – natural failure and induced failure. The natural failures are characterized by the lifetime distribution function $F_i(t)\,(i \in \Omega)$, and the induced failures are caused by failure of other components.
2. Type I failure interaction assumes that a natural failure of component i can cause the induced failure of component j with probability p_{ij}, and has no effect on component j with probability $q_{ij} = 1 - p_{ij}$. $i, j \in \Omega$. Apparently $p_{ii} = 1, q_{ii} = 0$. The interaction information can be integrated into the following matrix:
3. Failure dependence probability matrix (FDPM)

$$P = (p_{ij}) = \begin{bmatrix} 1 & p_{12} & \cdots & \cdots & p_{1,n-1} & p_{1,n} \\ p_{21} & 1 & \cdots & \cdots & p_{2,n-1} & p_{2,n} \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ p_{n-1,1} & p_{n-1,2} & \cdots & \cdots & 1 & p_{n-1,n} \\ p_{n,1} & p_{n,2} & \cdots & \cdots & p_{n,n-1} & 1 \end{bmatrix},$$
$$\times p_{ij} \in [0, 1], \quad \forall i, j \in \Omega$$

The parameters in matrix P can be estimated from test data by either parametric or nonparametric estimation techniques (Kapur, 2014). Particularly, regression model is an efficient method to deal with correlated failure time data (Kalbfleisch & Prentice, 2011).

Remarks. Failure interaction occurs either due to system mechanism or design problem, which decreases the system reliability and increases the warranty cost. Usually, the additional warranty cost induced by failure interaction is neglected by manufacturers when making warranty decisions. For a well-designed system, FDPM is a sparse matrix and the value of non-zero entry is relatively small. However, in practice, many systems do not perform as well as expected, and the warranty cost due to failure interaction cannot be neglected. Throughout this study, only Type I failure interaction is considered. The extension study on Type II or III failure interaction is left for future work.

2.3. Model specifications

Some critical definitions are made as follows:

1. Define $N_S(w)$ as the number of warranty renewals within warranty cycle T_S. $f_S(t)$ and $F_S(t)$ are denoted as the probability density function (pdf) and the cumulative distribution function (cdf) of the system failure time. $R_S(t) = 1 - F_S(t)$ is the system reliability function. $h_S(t) = f_S(t)/R_S(t)$ is the system hazard rate function. Similarly, $f_i(t), F_i(t), R_i(t)$, and $h_i(t)$ are defined as the pdf, cdf, reliability function, and hazard rate function of component $i, i \in \Omega$.
2. For a series system, let $T_i (i \in \Omega)$ be the natural failure time of component i. Define $p_i(w) \equiv \Pr[T_i \le Y_i, T_i \le w]$ and $\alpha_i(w) \equiv p_i(w)/F_S(w)$, where $Y_i = \min(T_j, \forall j \in \Omega, j \ne i)$, $\Omega = \{1, 2, \ldots, n\}$. $p_i(w)$ can be interpreted as the probability that a natural failure of component i occurs within warranty period w. Given a system failure within w, $\alpha_i(w)$ denotes the probability that the system failure is caused by the natural failure of component i. Lemma 1 presents the expressions of $p_i(w)$ and $\alpha_i(w)$.

Lemma 1. *For an n-component series system, we have*

$$p_i(w) = \int_0^w h_i(t) R_S(t)\,dt$$

$$\alpha_i(w) = \frac{1}{F_S(w)} \int_0^w h_i(t) R_S(t)\,dt$$

$$\sum_{i=1}^n p_i(w) = F_S(w), \quad \sum_{i=1}^n \alpha_i(w) = 1$$

See Appendix I for detailed proof.