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Decision Support

The implication of missing the optimal-exercise time of an American option

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ABSTRACT

The optimal-exercise policy of an American option dictates when the option should be exercised. In this paper, we consider the implications of missing the optimal exercise time of an American option. For the put option, this means holding the option until it is deeper in-the-money when the optimal decision would have been to exercise instead. We derive an upper bound on the maximum possible loss incurred by such an option holder. This upper bound requires no knowledge of the optimal-exercise policy or true price function. This upper bound is a function of only the option-holder's exercise strategy and the intrinsic value of the option. We show that this result holds true for both put and call options under a variety of market models ranging from the simple Black–Scholes model to complex stochastic-volatility jump-diffusion models. Numerical illustrations of this result are provided. We then use this result to study numerically how the cost of delaying exercise varies across market models and call and put options. We also use this result as a tool to numerically investigate the relation between an option-holder's risk-preference levels and the maximum possible loss he may incur when adopting a target-payoff policy that is a function of his risk-preference level.

1. Introduction

The *American put option* specifies a strike price K, an expiry time T, and affords the holder of the option the right to sell the underlying asset the option is written on for K at any time until and including T. Its European counterpart allows the holder to exercise the option only at T. At the time of exercise, the payoff of the option is the positive part of the difference between the strike price K and the current asset price (i.e., K less than asset price). The price of the option at any time is the maximum expected discounted payoff of the option over the remainder of its life.

To receive the maximum discounted payoff possible, the option holder needs to exercise the option optimally. Knowledge of the option price is essential to decide if the option should be exercised. If the immediate payoff exceeds the current price of the option, the holder should exercise the option, holding on to it otherwise. The optimal-exercise time is the first time that the immediate payoff, also known as the intrinsic value of the option, is equal to the price of the option. Essentially, there exists an optimal-exercise policy that dictates when the option holder should exercise the option.

The problem of pricing the American option is closely related to the determination of this optimal-exercise policy. In fact, the option pricing problem gives rise to a free-boundary problem in partial (integro) differential equations (PIDEs). Free-boundary problems are a class of problems in which the domain over which the PIDE is to be solved is not known a priori and needs to be solved simultaneously with the solution to the PIDE. The free boundary in the option pricing problem is the optimal-exercise policy. The solution to the PIDE is the option price function which describes the option price for a range of asset prices and times to expiry. Thus, solving the free-boundary problem yields the option price function and the optimal-exercise policy as this policy is the boundary of the domain over which the PIDE is solved.

The problem of pricing these options has received significant attention in literature. Under the assumption of constant volatility, Black and Scholes (1973) derive the celebrated Black–Scholes equation to compute the price of a European call option. The put-call parity can then be used to compute the price of the corresponding European put. Closed-form solutions for the price of European options have also been derived for market models which overcome the shortcomings of the Black–Scholes model, see for example, Heston (1993) and Madan, Carr, and Chang (1998). Closed-form solutions for European option

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prices may be computed in these cases because the exercise time is known with certainty.

Similar solutions do not exist for American options however, even under the Black–Scholes model (except for an American call option written on a stock that does not pay any dividends). The difficulty in computing such a solution arises as a result of the early-exercise feature of the American option. The lack of a closed-form solution has led to substantial research in developing approximations and computational schemes to price American options under a variety of market models. These schemes utilize simulation or numerical schemes such as finite difference and finite-element methods. Simulation-based methods typically compute the price of an option for a single time to expiry and asset price pair. The option price is then compared with the immediate payoff to decide if the option should be exercised (see for example Jin, Li, Tan, & Wu, 2013; Longstaff & Schwartz, 2001). Schemes based on finite difference and finite-element methods typically compute the entire price function. The optimal exercise policy can then be obtained either directly or as a post-processing step if the free-boundary problem was not directly solved. See, for example, Pressacco, Gaudenzi, Zanette, and Ziani (2008), for an overview and comparison of finite-difference and lattice-based methods.

Empirical evidence suggests that suboptimal exercise of American options occurs frequently in the markets. Diz and Finucane (1993) study exercise decisions in the S&P 100 market and conclude that many exercise decisions are inefficient (suboptimal). Bauer, Cosemans, and Eichholtz (2009) examine the impact of options trading on individual investor performance and find that investors incur significant losses on their options investment. These losses are indicative of suboptimal exercise of the options. Pool, Stoll, and Whaley (2008) also find that investors have incurred significant losses over a ten-year period by exercising call options suboptimally (including allowing the option to expire when it should have been exercised during its lifetime). Barraclough and Whaley (2012) study exercise decisions on put options and also find that a large number of options remain unexercised when they should have been, and that these suboptimal-exercise decisions have yielded significant losses to put option holders over a twelve-year period.

Suboptimal exercise of options is attributed to two main causes in literature, the first being model misspecification, which, as the name implies, refers to the incorrect specification of the underlying market model (for example assuming constant volatility when volatility is in fact stochastic). Model misspecification results in investors incorrectly pricing options, obtaining an incorrect exercise policy and consequently making a suboptimal-exercise decision. Longstaff, Santa-Clara, and Schwartz (2001) study the effect of using single-factor models to make exercise decisions on swaptions when the underlying term structure is actually driven by several factors. The authors conclude that the total possible present value of the costs of following single factor strategies could be several billion dollars, even when the single factor is re-calibrated frequently. Chockalingam and Muthuraman (2011) demonstrate that pricing American put options assuming a constant volatility when the volatility of the underlying asset price is stochastic leads to mispricing of the American option, even when the constant volatility is set equal to the mean level of the stochastic process describing the evolution of volatility. As mentioned before, this would lead to misidentification of the optimal-exercise policy value and therefore suboptimal-exercise decisions. Irrational investor behavior is cited as the second cause for suboptimal-exercise decisions of options. Classical option pricing theory computes option prices and exercise policies assuming that investors are rational agents. Empirical studies, however, have demonstrated that investors do indeed exercise options irrationally. Overdahl and Martin (1994) and Poteshman and Serbin (2003) find evidence of this in exchange traded call options. Engstrom (2002) studies exercise decisions on call options in the Swedish stock market and finds evidence of the same. Finucane (1997) also studies empirically exercise decisions on call options, comparing these results to results obtained under the assumption that there is no friction present in the market. The author finds that 20 percent of exercises are not optimal, and that even if a large number of these irrational trades can be explained by the presence of transaction costs, a significant number of exercises still appear to be irrational.

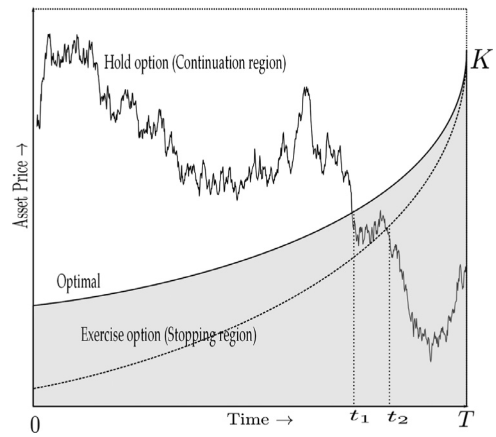

Fig. 1. Illustration of delaying exercise.

The literature on characterizing the cost of suboptimal exercise is relatively scarce. In Ibáñez and Paraskevopoulos (2010), the authors study the sensitivity of the American option to suboptimal-exercise policies by considering policies that advance exercise, and those that delay exercise. In the put option case, the advancing (delaying) exercise corresponds to the exercise boundary lying above (below) the optimal-exercise boundary. They find that the cost of suboptimal exercise is a function of the gamma of the option at the optimal-exercise boundary and the bias in the suboptimal-exercise policy (difference between the optimal and sub-optimal boundaries).

In this paper, we consider the cost of delaying exercise of an American option. As mentioned above, in the put option case this corresponds to exercise policies that dictate the holder should hold on to the option till it is deeper in the money when the optimal action would have been to exercise the option. Naturally, the optimal action, along with the optimal-exercise policy, is unknown to us. Such suboptimal policies delay exercise of the option. This situation is illustrated below in Fig. 1 for the classic Black–Scholes model. In the figure, the stopping region (where the option should be optimally exercised is shaded. The optimal-exercise policy dictates that the option should be exercised at time t_1 when the asset price strikes the policy from the above). The suboptimal-exercise policy prescribes waiting till time t_2 to exercise the option. While the structure of the exercise policies will change when the underlying asset price is assumed to be modeled by different processes, this concept of delaying exercise will remain unchanged throughout the paper.

By decomposing the price of the American put option into the price of the corresponding European put and an early exercise premium, Carr, Jarrow, and Myneni (1992) derive an expression for the delayed-exercise premium, i.e., the additional benefit gained by delaying exercise till the optimal-exercise time. This perspective, and consequently, the delayed-exercise premium, differ significantly from our approach and the upper bound we compute. In our case, we consider delaying the exercise of the put option past the optimal-exercise time. As such, the premium from Carr et al. (1992) will not coincide with the upper bound on the cost of delaying exercise that we compute later in the paper.

Ibáñez and Paraskevopoulos (2010) note that computing the cost of delaying exercise is more difficult than computing its advancing exercise counterpart as the cost of delaying exercise depends on the entire suboptimal-exercise policy while the cost of advancing exercise