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Decision Support

A discontinuous mispricing model under asymmetric information

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ABSTRACT

We study a discontinuous mispricing model of a risky asset under asymmetric information where jumps in the asset price and mispricing are modelled by Lévy processes. By contracting the filtration of the informed investor, we obtain optimal portfolios and maximum expected utilities for the informed and uninformed investors. We also discuss their asymptotic properties, which can be estimated using the instantaneous centralized moments of return. We find that optimal and asymptotic utilities are increased due to jumps in mispricing for the uninformed investor but the informed investor still has excess utility, provided there is not too little or too much mispricing.

1. Introduction

Asymmetric information models assume that there are two types of investors in the market: informed and uninformed. The informed investors (e.g., institutional investors with internal research capabilities) observe both fundamental and market prices, while uninformed investors (e.g., retail investors who rely on public information in order to make investment choices) observe market prices only. The uninformed investors are viewed as liquidity traders or hedgers. The prevalence of informed traders affects liquidity, transaction costs, and trading volumes. The informed investors partially reveal information through trades, which can cause higher permanent price changes. Asymmetric information asset pricing models rely on a noisy rational expectation equilibrium in which prices only partially reveal the informed investor's information. Many empirical studies confirm that information asymmetry is priced and imply that liquidity is a primary channel that links information asymmetry to prices (see, e.g., Admati, 1985; Easley & O'Hara, 2004; Grossman & Stiglitz, 1980; Hellwig, 1980; Kelly & Ljungqvist, 2012; Wang, 1993).

Mispricing is the difference between the asset's market price and fundamental value. The fundamental value can be defined as the market price that would prevail if all the market participants were perfectly informed investors. Because of the mean-reverting nature of the mispricing process, it is typically modelled by a continuous Ornstein–Uhlenbeck (O–U) process, while the price of the risky asset is usually modelled by a continuous geometric Brownian motion (see, e.g., Buckley, Brown, & Marshall, 2012; Guasoni, 2006; Wang, 1993). The mean-reverting speed or equivalently, the mean reverting-time, is a proxy for mispricing. Mean-reversion[1] is well-documented in the empirical financial literature and applies to asset returns, stock prices, currencies/exchange rates, interest rates, commodities, indexes, stock index futures, and options.

This paper addresses how asymmetric information, mispricing, and jumps in both the price of a risky asset and its mispricing affect the optimal portfolio strategies and maximum expected utilities of two distinct classes of rational long-horizon investors in an economy where preference is logarithmic. For the purpose of this exposition, we take the risky asset to be stock, but the model can be applied to any asset (e.g. currencies) where prices are mean-reverting. The investors assume that the risky asset has a fundamental or true value (for example, expected discounted future dividends) as well as a market price. When prices move away from its fundamental value, they always revert to it (see, e.g., LeRoy & Porter, 1981; Poterba & Summers, 1988; Shiller, 1981; Summers, 1986). What do we mean by fundamental value? Allen, Morris, and Postlewaite (1993) posit that the fundamental value of an asset is the present value of the stream of the market value of dividends or services generated by this asset. The fundamental value can also be defined as the market price that would prevail if the cost of gathering and processing information is zero for

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[1] The literature related to mean-reversion in asset prices is very large and includes stocks: Poterba and Summers (1988), Depenya and Gil-Alana (2002), Gropp (2004), Mukherji (2011, 2012); currencies: Engel and Hamilton (1990), Cheung and Lai (1994), Sweeney (2006), Serban (2010); indices: Miller, Muthuswamy, and Whaley (1994), Balvers, Wu, and Gilliland (2000), Balvers and Wu (2006), Caporale and Gil-Alana (2008), Kim, Stern, and Stern (2009), Chen and Kim (2011), Spierdijk, Bikker, and van den Hoek (2012), Malin and Bornholt (2013); index futures: Monoyios and Sarno (2002), Miao, Lin, and Chao (2014).

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all investors. Any of the definitions above will suffice. However, we adopt the caveat in Allen et al. (1993) and take fundamental value to mean the value of an asset in normal use, as opposed to some value it may have as a speculative instrument.

Because mispricing is the difference between the asset's market price and its fundamental value, if the fundamental value or asset price jumps, then it follows that the mispricing will jump as well, provided the jump components are not identical and cancel each other. This is particularly evident in the case of independently driven jump processes (see, e.g., Applebaum, 2004). All recent studies (e.g., Buckley et al., 2012; Buckley, Long, & Perera, 2014; Guasoni, 2006) assume that mispricing is a continuous O–U process. However, in this paper, mispricing is no longer purely continuous. Instead, it jumps and is driven by a mean-reverting O–U process which has a continuous component as well as a discontinuous component generated by a pure-jump Lévy process. As in Buckley et al. (2014), the price of the risky asset is still subject to Levy jumps. We solve this model for both investors and present explicit formulas for their optimal portfolios and maximum expected logarithmic utilities under reasonable assumptions.

Portfolio allocation problems have been extensively studied since the seminal work of Markowitz (1952). Merton's (1971) model is the benchmark of optimal asset allocation in the continuous-time framework. There is a rich plethora of portfolio optimization papers[2] in various settings. We refer to Buckley et al. (2012, 2014) for related literature review and discussion on recent progress on this topic.

In this paper, we find that the optimal portfolio of each investor contains excess risky asset that depends on the Lévy measures of both jump processes (asset price and mispricing), the diffusive volatility, and the level of mispricing, as represented by the mean-reversion speed or time. Under quadratic approximation of the portfolios, we show that excess asset holdings are dependent on the first two instantaneous centralized moments of return (see, e.g., Cvitanic et al., 2008). In addition, the maximum expected utility from terminal wealth for each investor is increased by the presence of jumps in the mispricing. This suggests that investors are better off when mispricing jumps than when it changes continuously.

Further, we show that the uninformed investor obtains excess utility from jumps in mispricing. However, as proven in prior studies by Guasoni (2006) and Buckley et al. (2012, 2014), the informed investor still has positive excess utility over the uninformed investor. This implies that the informed investor gets more utility from the diffusive component driving the asset price process. We also show that the asymptotic excess optimal expected utility from terminal wealth of the informed investor has a similar structure to that presented in Guasoni (2006) and Buckley et al. (2012, 2014), but is increased by a factor directly attributed to the jumps in the mispricing. As in the case of jumps in stock price only, the mean-reversion rate λ is replaced by a smaller adjusted mean-reversion rate $\tilde{\lambda}$, which depends on the volatility of the asset price only.

There is also an equivalent representation of excess utility by way of the mean-reversion speed $\hat{\lambda}$ of an adjusted continuous O–U mispricing process which also depends on the volatility of the long-run asset price. In this framework, the excess optimal expected utility of the informed investor is greater (less) than its continuous Merton (1971) geometric Brownian motion counterpart if the product of the diffusive variance and the quadratic variation of the mispricing process is greater (less) than the second instantaneous centralized moment of return of the jump component of the asset price process. Notwithstanding the presence of asymmetric information, mispricing and jumps, our results show that it still pays to be more informed in the long-run, unless there is too little or too much mispricing. Moreover, our results nest those contained in Guasoni (2006) and Buckley et al. (2014).

The practical, financial, economic and operational implication of this paper is that when asymmetric information and jumps exist in both asset price and mispricing, informed and uninformed long-horizon investors will maximize their expected logarithmic utilities from terminal wealth by holding portfolios that contain excess asset holdings which depend not only on the level of information asymmetry and preference, but also on the nature and frequency of the jumps in both asset price and mispricing as dictated by the governing Lévy measures. To the best of our knowledge, this paper is the first to study the utility and portfolio implications for the risky asset by investors when asymmetric information, logarithmic preferences, jumps in asset price and mispricing are intertwined, and therefore contributes to the finance and operational research literature.

Our model is related to heterogeneous agents models (HAM), where investors observe the same information but have different beliefs. However, there are two important distinctions. First, switching is not allowed between investor classes, i.e., an uninformed investor is not allowed to become informed, and vice versa. Second, one investor knows more than the other (see, e.g., Chiarella, Dieci, & He, 2009 and He, 2012 for a review of the HAM literature).

As in Buckley et al. (2014), our model is different from insider trading models[3] which use enlargement of filtrations to obtain the optimal portfolio and utility of the insider trader/informed investor. In contrast to insider trading models, we specify the price dynamics of the informed investor in the larger filtration, and then obtain the dynamics for the uninformed investor by contracting (or restricting) the larger filtration. We then compute and compare optimal portfolios and expected logarithmic utilities for each investor relative to their respective filtrations.

The remainder of the paper is organized as follows. The model is introduced in Section 2, including information flows/filtration and asset price dynamics of investors. In Section 3, expected utilities are maximized, and optimal portfolios are computed and estimated using instantaneous centralized moments of returns. Asymptotic and excess utilities are presented in Section 4. We conclude in Section 5 by giving directions of possible future research. All proofs are given in Appendix A.

2. The discontinuous mispricing model

The economy consists of two assets—a risk-free asset **B**, called bank account or money market (or U.S. Treasury bill), and a risky asset S, called stock. The risk-free asset earns a continuously compounded risk-free interest rate r_t, and has price $\mathbf{B}_t = \exp\left(\int_0^t r_s\,ds\right)$. The continuous component of the stock's percentage appreciation rate is μ_t, at time $t \in [0, T]$, where $T > 0$ is the investment horizon. The stock is subject to volatility $\sigma_t > 0$. The market parameters are $\mu_t,\ r_t,\ \sigma_t,\ t \in [0, T]$, and are assumed to be **deterministic** functions. The stock's **Sharpe ratio** or **market price of risk** $\theta_t = \frac{\mu_t - r_t}{\sigma_t}$ is square integrable. The risky asset has price S, and lives on a probability space $(\mathbf{\Omega}, \mathcal{F}, \mathbf{P})$ on which is defined two independent standard Brownian motions $W = (W_t)_{t\geq 0}$ and $B = (B_t)_{t\geq 0}$. The stock is viewed by investors in disjoint classes populated by uninformed and informed investors, indexed by $i = 0$ and $i = 1$, respectively. Investors have filtrations $\mathcal{K}_t^i$

[2] See Mansini and Speranza (1999), Kellerer, Mansini, and Speranza (2000), Soyer and Tanyeri (2006), Celikyurt and Ozekici (2007), Corazza and Favaretto (2007), Cvitanic, Polimenis, and Zapatero (2008), Lin and Liu (2008), Canakoglu and Ozekici (2010), Fu, Lari-Lavassani, and Li (2010), Huang, Zhu, Fabozzi, and Fukushima (2010), Yu, Takahashi, Inoue, and Wang (2010), and Buckley et al. (2012, 2014).

[3] See, e.g., Pikovsky and Karatzas (1996), Amendinger, Imkeller, and Schweizer (1998), Amendinger, Becherer, and Schweizer (2003), Ankirchner, Dereich, and Imkeller (2006), Ankirchner and Imkeller (2007), who solve this problem for logarithmic and power utilities. Grorud (2000) studies insider trading in a discontinuous market, while Goll and Kallsen (2003) give a complete explicit characterization of log-optimal portfolios without constraints.