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Discrete Optimization

A hybrid metaheuristic algorithm for the multi-depot covering tour vehicle routing problem

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ABSTRACT

We propose a generalization of the multi-depot capacitated vehicle routing problem where the assumption of visiting each customer does not hold. In this problem, called the Multi-Depot Covering Tour Vehicle Routing Problem (MDCTVRP), the demand of each customer could be satisfied in two different ways: either by visiting the customer along the tour or by "covering" it. When a customer is visited, the corresponding demand is delivered at its location. A customer is instead covered when it is located within an acceptable distance from at least one visited customer from which it can receive its demand. For this problem we develop two mixed integer programming formulations and a hybrid metaheuristic combining GRASP, iterated local search and simulated annealing. Extensive computational tests on this problem and some of its variants clearly indicate the effectiveness of the developed solution methods.

1. Introduction

The vehicle routing problem (VRP) is one of the most widely studied problems in the field of combinatorial optimization. VRP literature dates back to 1959 when Dantzig and Ramser (1959) introduced it for the first time as the "truck dispatching problem". Generally, VRP is aimed at servicing a given set of customers using a set of vehicles located at a central depot. Several objectives have been reported for the VRP among which we can mention the minimization of the routing cost (time) and the minimization of the total fixed and variable costs. For a complete review of the different VRP variations we refer the interested readers to the books Cordeau, Laporte, Savelsbergh, and Vigo (2007, chap. 6), Golden, Raghavan, and Wasil (2008), Toth and Vigo (2002) and the survey papers by Eksioglu, Vural, and Reisman (2009) and Laporte (2007, 2009). In the classical capacitated VRP (CVRP), we are given a fleet of homogenous capacitated vehicles located at a central depot. The goal of the CVRP is to construct a set of vehicle routes having the minimum total cost while satisfying the entire demand of the customers. Each vehicle starts and ends its route at the depot while the total demand carried by each vehicle cannot exceed its given capacity. Several heuristics, metaheuristics and exact methods have been proposed for the solution of the CVRP in the literature (see, e.g. Baldacci, Hadjiconstantinou, & Mingozzi, 2004; Baldacci & Mingozzi, 2009; Chen, Huang, & Dong, 2010; Cordeau et al., 2007, chap. 6; Toth & Vigo, 2002). In contrast to the classical VRP in which a single depot is available, in the multi-depot VRP (MDVRP) we are given several depots, each equipped with a given number of vehicles (see, e.g. Chao, Golden, & Wasil, 1993; Laporte, Nobert, & Arpin, 1984; Liu, Jiang, Liu, & Liu, 2011; Tillman, 1969; Toth & Vigo, 2002). Several methods have been developed to solve the MDVRP including a branch and bound exact method (Laporte, Nobert, & Taillefer, 1988), heuristics (Chao et al., 1993; Gillett & Johnson, 1976; Raft, 1982; Sumichras & Markham, 1995) and metaheuristics (such as genetic algorithm (Ho, Ho, Ji, & Lau, 2008; Vidal, Crainic, Gendreau, Lahrichi, & Rei, 2012), adaptive large neighborhood search (Pisinger & Ropke, 2007), variable neighborhood search (Polacek, Hartl, Doerner, & Reimann, 2004), tabu search (Renaud, Laporte, & Boctor, 1996) and ant colony optimization (Yu, Yang, & Xie, 2010)). A recent survey on the MDVRP and its variants is performed in Tiantang, Zhibin, Ran, and Shujun (2011).

In many practical applications due to restrictions on time, budget, resource availability (e.g., the number or the capacity of the vehicles) or unavailability of roads reaching a specific customer, it is not possible to visit all the customers with the vehicles routes. To cope with some of these situations, the option of *covering* has been introduced in the literature. In this case the demand of unvisited (covered) customers could be delivered at a place located within an acceptable walking distance from that. The covering salesman problem (CSP) is one of the first routing problems in which visiting each of the customers on the tour is not necessary (see, Current & Schilling, 1989).

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In this problem we are given a single vehicle and a set of customers. The goal is to construct a minimum length Hamiltonian cycle over a subset of the customers in such a way that unvisited locations lay within a pre-specified covering distance from at least one visited customer. Several solution methods have been presented in the literature for the CSP and its variants (Arkin & Hassin, 1994; Golden, Naji-Azimi, Raghavan, Salari, & Toth, 2012; Salari & Naji-Azimi, 2012).

Gendreau, Laporte, and Semet (1997) introduced the covering tour problem (CTP) in which the set of vertices is divided into two groups (i.e. $N = N_1 \cup N_2$). Set N_1 includes the vertices that can be visited and contains a set T of vertices that must be visited. In addition, set N_2 includes the vertices that must be covered by the tour. The goal of the CTP is to construct a minimum length Hamiltonian cycle over the vertices in T and possibly a subset of vertices in $N_1 \setminus T$, in which all vertices in N_2 are covered. Hachicha, Hodgson, Laporte, and Semet (2000) developed a multi vehicle variant of the CTP with several practical applications, including the design of the routes for mobile healthcare delivery teams. In this problem the goal is to design m Hamiltonian cycles over a subset of vertices to visit the vertices in T and cover all of the vertices in N_2. The authors proposed a mathematical formulation and three heuristic algorithms for the introduced problem. Moreover, Lopes, Souza, and da Cunha (2013) developed a branch-and-price algorithm for this problem. Finally, branch-and-cut and metaheuristic algorithms were proposed by Bostel et al. (Hà, Bostel, Langevin, & Rousseau, 2013).

The covering issue has a wide variety of applications in the real world. As an example, we can find the combination of routing and covering concepts in problems arising in emergency situations such as earthquake, flood and tsunami (Altay & Green, 2006; Caunhye, Nie, & Pokharel, 2012; De La Torre, Dolinskaya, & Smilowitz, 2012). Nolz, Doerner, Gutjahr, and Hartl (2010) defined a multi-objective covering tour problem to distribute water to people in an area affected by a disaster. The objectives considered in this problem are the minimization of the total distance travelled by the covered customers to reach their nearest visited vertex, the number of the customers unable to reach a visited customer within a pre-specified maximum distance, the tour length, and the latest arrival time at a customer. Doerner, Focke, and Gutjahr (2007) described a multi-objective combinatorial optimization problem which is applicable in the healthcare management. The goal of their problem is to design a single tour through a subset of vertices by taking three objective functions into account: (1) improving the economic efficiency of the tour, (2) minimizing the average distances traveled by the unvisited people to reach their nearest tour stops and (3) minimizing the population percentage unable to reach a tour stop within a pre-specified maximum travel time. Another application of covering to healthcare is described in Hodgson, Laporte, and Semet (1998). Finally, Naji-Azimi, Renaud, Ruiz, and Salari (2012) modeled a generalization of the covering tour problem in which the customers demand is fulfilled by some satellite distribution centers located within a predefined distance from their domiciles.

In this paper, we propose a generalization of the MDVRP including the covering option, called the Multi-Depot Covering Tour Vehicle Routing Problem (MDCTVRP). Here, the demand of each customer could be satisfied either directly, by being visited on the tour or indirectly, by being covered by the tour, i.e., when its location is within a given *covering distance* of at least one visited customer. In other words, our problem is a combination of the MDVRP and CSP problems. Among the many practical applications of MDCTVRP we mention the distribution of goods to the people in an area affected by a disaster. In such a situation, the humanitarian services could be provided by different depots and, because of the limitation in time resource we cannot visit all customers on the routes. In addition to the routing cost occurred by visiting the customers on the vehicles' routes, we introduce a covering cost which is proportional to the distance travelled by the covered customers to reach their corresponding allocated nearest customer visited on a route. To the best of our knowledge this problem has not been previously addressed by the literature. An illustrative example of the studied problem is given in Fig. 1. This sample network contains 2 depots, 74 customers and 2 vehicles per depot. Overall, 30 customers are visited by the tours which are used to cover the demand of the unvisited customers.

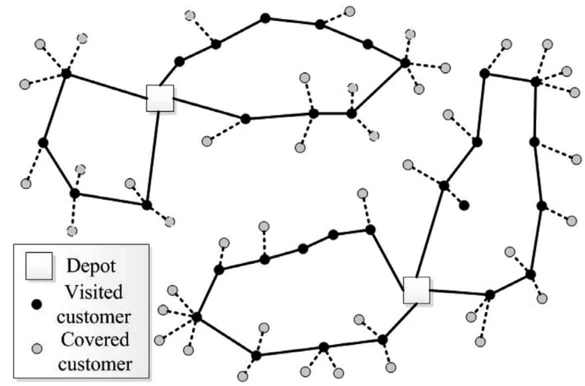

Fig. 1. An illustrative example of the problem.

The paper is organized as follows. The formal description of the introduced problem is provided in Section 2 where two mixed integer linear programming models are introduced. Details of the proposed hybrid metaheuristic algorithm are presented in Section 3. In Section 4 we report the results of computational tests of the proposed algorithms on MDCTVRP and some of its variants. Finally, some conclusions are drawn in Section 5.

2. Problem description

In MDCTVRP we are given a directed graph $G = (N, A)$ in which $N = N_C \cup N_D$ is the set of vertices, and $A = \{(i, j) | i, j \in N\}$ is the set of arcs. More precisely, $N_C = \{1, 2, \ldots, n_c\}$ represents the set of customer vertices where each $i \in N_C$ has a pre-specified demand, $d_i > 0$, to be met by exactly one vehicle. Moreover, $N_D = \{1, 2, \ldots, n_d\}$ is the set of depots from which the vehicles start their trips. Each arc $(i, j) \in A$ is associated with a non-negative routing cost c_{ij}, equal to the cost of traversing arc (i, j) by the vehicles. In MDCTVRP it is not necessary that each customer is visited by a vehicle and the unvisited customers should be within an acceptable distance from at least one visited customer. To this end we define $\pi = [\pi_{ij}]_{n_c \cdot n_c}$ as the covering matrix, where $\forall i, j \in N_C$, $\pi_{ij} = 1$ if and only if customer i is located within a pre-specified distance from customer j. Note that to favor feasibility we assume that for each $i \in N_C$ there is at least one $j \in N_C$ such that $\pi_{ij} = 1$. For each $i, j \in N_C$ the non-negative cost c'_{ij} represents the allocation cost of customer i to the visited customer j. We assume that a limited set $P = \{1, 2, \ldots, p\}$ of capacitated vehicles is available, where each vehicle $v \in P$ has capacity Q. The assumption is that the capacity of each depot $k \in N_D$ is limited and equals to H. Finally, at each depot k it is located a given set of vehicles represented by $P_k = \{1, 2, \ldots, p_k\}$ where P_k constitute a partition of P.

We have developed two mathematical models, namely one flow-based and one node-based formulation for MDCTVRP that will be used in the computational evaluations. Before getting into the details of each model, we define the variables which are common to both formulations. In particular, we use two sets of binary decision variables:

$$x_{ij}^v = \begin{cases} 1 & \text{if arc } (i, j) \in A \text{ is traversed by vehicle } v \in P, \\ 0 & \text{otherwise.} \end{cases}$$