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Discrete Optimization

# A Jackson network model and threshold policy for joint optimization of energy and delay in multi-hop wireless networks ☆

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ABSTRACT

This paper studies the joint optimization problem of energy and delay in a multi-hop wireless network. The optimization variables are the transmission rates, which are adjustable according to the packet queueing length in the buffer. The optimization goal is to minimize the energy consumption of energy-critical nodes and the packet transmission delay throughout the network. In this paper, we aim at understanding the well-known decentralized algorithms which are threshold based from a different research angle. By using a simplified network model, we show that we can adopt the semi-open Jackson network model and study this optimization problem in closed form. This simplified network model further allows us to establish some significant optimality properties. We prove that the system performance is monotonic with respect to (w.r.t.) the transmission rate. We also prove that the threshold-type policy is optimal, i.e., when the number of packets in the buffer is larger than a threshold, transmit with the maximal rate (power); otherwise, no transmission. With these optimality properties, we develop a heuristic algorithm to iteratively find the optimal threshold. Finally, we conduct some simulation experiments to demonstrate the main idea of this paper.


## 1. Introduction

Energy efficiency is one of the most important requirements of wireless networks since most of the mobile end users are powered by battery (Montemanni, Leggieri, & Triki, 2008; Shakkottai, Rappaport, & Karlsson, 2003). On the other hand, low latency of data transmission is another important requirement for many real-time applications. Here, the data delay includes only the transmission delay and the queueing delay, the propagation delay and the precessing delay are omitted since they are very small in our scenario. High transmission rate can reduce the end-to-end delay with the cost of consuming more energy. Therefore, it is a fundamental problem that how to schedule the packet transmission to optimize the transmission delay and the energy consumption in a wireless network (Berry & Gallager, 2002; Ren & Meng, 2009).

Ad-hoc is a preferred communication mode for wireless networks when the end users are mobile. The decentralized control scheme is suitable for ad-hoc networks since it has a good scalability and flexibility. In this paper, we study a hybrid network. That is, the network consists of two types of communication nodes, one with adequate energy supply and the other with limited energy. We call it *partial energy-critical* wireless network. This partial energy-critical wireless network exists in the real world. For example, the hybrid ad-hoc network deployed in desert may have some nodes with infrastructure and power supply, and other nodes have neither infrastructure nor power supply (Ben Salem, Buttyan, Hubaux, & Jakobsson, 2006). Another example is that a heterogeneous wireless sensor network may have some sensor nodes whose remaining energy is relatively low (Mhatre, Rosenberg, Kofman, Mazumdar, & Shroff, 2005). The goal is to optimize the energy consumption of these nodes in order to extend their life times without largely increasing the data delay of the entire network. Fig. 1 is an example of the network topology of a partial energy-critical wireless network.

The energy and delay optimization problem in wireless networks has been studied in the literature. Some of the studies discuss the optimization of energy consumption with a delay constraint (Chen, Neely, & Mitra, 2007; Qiu, Bai, & Xue, 2014; Zhong & Xu, 2008). As a comparison, some other studies propose to minimize the delay under a constraint of energy consumption (Goyal, Kumar, & Sharma, 2008; Yang & Ulukus, 2010). However, all of the studies above consider only a single node as the optimization target. In a multi-hop wireless network, communication nodes are interconnected and the dynamics of nodes have mutual influences. Therefore, it is necessary to consider

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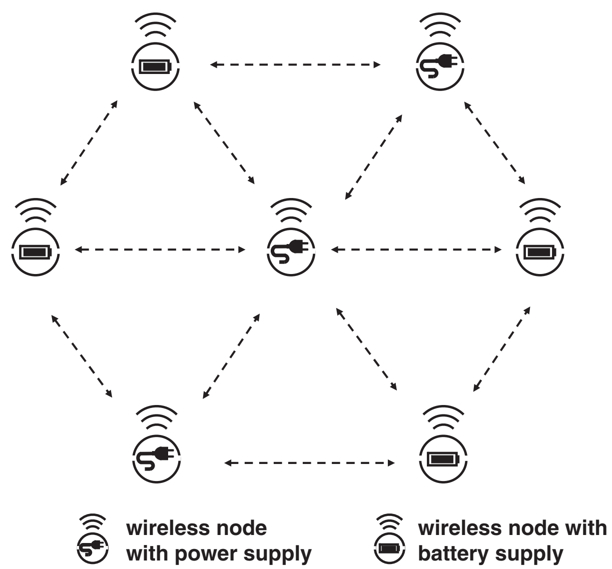


**Fig. 1.** Example topology of a partial energy-critical wireless network.

the mutual influence of nodes during the energy and delay optimization (Neely, 2007; Rajan, 2007). Cross-layer optimization is a widely adopted way to study this kind of problems (Cruz & Santhanam, 2003; Cui, Madan, Goldsmith, & Lall, 2007; Lin, Shroff, & Srikant, 2006; Liu, Xia, Shroff, & Sherali, 2013; Xiong, Li, Eryilmaz, & Ekici, 2011) and different approaches are used to optimize the scheduling policy, such as the gradient based approach (Liu et al., 2013), stability analysis approach (Neely, 2007), or other queueing model based analytical approaches (Xiong et al., 2011). With a queueing network model, Xia and Shihada (2014) study the joint optimization of energy and delay in a multi-hop wireless network with the full observation of system buffer status and channel status. However, all these studies are based on a centralized control scheme. The network needs a central controller to acquire the full system information and to determine the node's scheduling policy. Such a centralized control scheme does not have a good scalability and flexibility.

As a comparison, decentralized control is a preferred regime for wireless networks, and it has the advantages on scalability and flexibility (Abouei, Ebrahimi, & Khandani, 1969; Kho, Rogers, & Jennings, 2009). How to develop a decentralized optimization approach under a multi-hop network scenario is an important topic in the research field of wireless networks (Lin, Lin, & Shroff, 2010; Shakkottai et al., 2003). Moreover, there are some other works studying the rate control and scheduling problem in networking systems from other viewpoints. The work by Xia and Shihada (2013a) studies the optimality properties of service rate control problem in a closed Jackson network. But this paper does not discuss the optimization algorithm to find the optimal policy. The works by Eryilmaz and Srikant (2007) and Lin and Shroff (2004) conducted good studies on jointly considering the congestion control and the resource scheduling problem. But these studies do not take the energy efficiency as their first optimization target in a multi-hop network scenario.

In this paper, we aim at using a closed-form analysis to study the decentralized optimization problem of energy and delay in multi-hop wireless networks, with the cost of a simplified network model. The transmission rates of nodes are adjustable and the optimization goal is to minimize the average energy consumption and the data delay of the entire network. Every node observes only their own local information including the buffer status and the system reward to determine their own transmission strategy. The network model is simplified under some assumptions. For example, we assume that the channel is stable and the interference is not considered since this issue is not severe after adopting proper physical or data-link layer techniques (such as the synchronization and TDMA).

With proper assumptions, we use a simplified network model, semi-open Jackson network, to formulate this multi-hop wireless network. The associated optimization problem of energy and delay is formulated as a Markov decision process (MDP) (Guo & Hernandez-Lerma, 2009; Puterman, 1994). We utilize the special structure of product-form solution of Jackson networks to study this problem in closed form. Some interesting optimality properties are derived as follows. The system performance is monotonic w.r.t. the transmission rate. The optimal scheduling strategy has a threshold form. That is, when the queue length in the buffer is larger than a threshold, the optimal transmission rate is maximal; otherwise, it is minimal. This indicates that a "bang-bang" control (Artstein, 1980; Bellman, Glicksberg, & Gross, 1956) is an optimal control for this problem. The optimality of threshold-type policy greatly reduces the optimization complexity. We further develop a heuristic iterative algorithm to find the optimal threshold-type policy. This algorithm does not require the full system information and it can be implemented in an online manner. This paper is substantially extended based on our previous conference paper (Xia & Shihada, 2013b) and the new contents include the online estimation algorithm, the detailed proofs of optimality properties, the analysis of numerical experiments, etc. The limitation of this paper mainly exists in the simplification of the network model. Since our emphasis is the closed-form analysis of the threshold-type policy, some assumptions are required to simplify the network model. If we consider more practical issues, such as the interference and the fading channel, we need more sophisticated investigations in the future.

The remainder of the paper is organized as follows. In Section 2, we give a detailed description of our problem and formulate it with a simplified network model. In Section 3, we utilize the special structure of Jackson network to obtain the performance difference formula of this optimization problem. We also derive some optimality properties in this section. In Section 4, we propose a heuristic and iterative algorithm to distributely optimize the threshold-type policy. The detailed algorithm and its online implementation are also discussed. In Section 5, we discuss some limitations of our simplified network model and the optimization approach. In Section 6, we conduct some numerical experiments to demonstrate the effectiveness of our approach. Finally, we conclude the paper in Section 7.

## 2. Problem description and model formulation

Consider a partial energy-critical wireless network. The network consists of $M$ communication nodes. The number of energy-critical nodes is denoted as $M_1$ and the number of energy-noncritical nodes is $M - M_1$. Without loss of generality, we assume that the first $M_1$ indexed nodes are energy-critical. That is, server $i$ is energy-critical and server $j$ is energy-noncritical, $i = 1, 2, \ldots, M_1, j = M_1 + 1, \ldots, M$. We assume that the data packet to be transmitted is independently generated at each node. The data generation process is assumed as a Poisson process and its rate at node $i$ is denoted as $\lambda_i, i = 1, 2, \ldots, M$. The network usually has different types of data packets (packets for HTTP, FTP, video, audio, control protocols, etc.) and different types of packets have different packet sizes. We assume that the packet size is exponentially distributed with a unit mean. Thus, the transmission time of each packet obeys an exponential distribution. Since the network has multi-hop transmission routes, the generated packet will be relayed among nodes according to a routing protocol. We assume that the routing table is relatively stable and the statistics of the routing traffic has the following distribution. When a packet arrives at node $i$, this packet will be relayed to adjacent node $j$ with routing probability $q_{ij}, i, j = 1, 2, \ldots, M$. On the other hand, a packet may arrive at its final destination, say node $i$, with probability $q_{i0}$ and disappear from the network, $i = 1, 2, \ldots, M$. Obviously, we have $\sum_{j=0}^{M} q_{ij} = 1$ for all $i = 1, 2, \ldots, M$. This assumption of routing probability is reasonable