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Periodic review and continuous ordering

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ABSTRACT

Many inventory control studies consider either continuous review and continuous ordering, or periodic review and periodic ordering. Mixtures of the two are hardly ever studied. However, the model with periodic review and continuous ordering is highly relevant in practice, as information on the actual inventory level is not always up to date while making ordering decisions. This paper will therefore treat this model. Assuming zero fixed ordering costs, and allowing for a non-negative lead time and a general demand process, we first consider a one-period decision problem without salvage cost for inventory remaining at the end of the period. In this setting we derive a base-line optimal order path, described by a simple newsvendor solution with safety stocks increasing towards the end of a review period. We then show that for the general, multi-period problem, the optimal policy in a period is to first arrive at this path by not ordering until the excess buffer stock from the previous review period is depleted, then follow the path by continuous ordering, and stop ordering towards the end to limit excess stocks for the next review period. An important managerial insight is that, typically, no order should be placed at a review moment, although this may seem intuitive and is also the standard assumption in periodic review models. We illustrate that adhering to the optimal ordering path instead can lead to cost reductions of 30–60 percent compared to pure periodic ordering.

1. Introduction

In the inventory control literature, the focus is often on two extreme cases: either periodic stock review and periodic ordering at that same review point, or continuous stock review and continuous order possibilities. See e.g. Axsäter (2006) and Silver, Pyke, and Peterson (1998) for discussions of such models. Mixtures of both extremes are hardly ever studied. For continuous review and periodic ordering this is not surprising, since in a single-item setting the optimal policy will be equal to the pure periodic review solution with review periods equal to the time inbetween ordering points. Therefore, the sole contributions to the literature in this setting consider multi-item models. Some work has also been done on the situation of continuous review and periodic ordering, in the specific case where multiple products are jointly replenished from the same supplier to achieve cost savings. Some of the first, concrete steps here were made by Goyal (1974), who introduced an optimal algorithm for this problem. Since then a number of others have also studied this so-called "Joint Replenishment Problem". Recently, Roushdy, Sobhy, Abdelhamid, and Mahmoud (2011) proposed an iterative method for a specific review structure and Zhang, Kaku, and Xiao (2012) studied this problem under correlated demands.

Interestingly and perhaps surprisingly, the other mixture, periodic review and continuous ordering, has never been studied to the best of our knowledge, at least not with "truly" continuous ordering. There have been a number of contributions where orders are allowed at a number of predefined times during a period. Two decades ago, Flynn and Garstka (1990) already formulated a model and according policies where orders are allowed to be placed at the start of sub-periods of equal length during a review period. Chiang (2001) proposes order splitting in a periodic review framework. That is, at the start of a period an order is placed, and this order arrives in batches with fixed interarrival times in the current period. This method provides a holding cost advantage, which is shown by minimizing costs under a service level constraint.

However, as mentioned before, none of the previous periodic review studies considers continuous ordering, i.e. potential ordering at any point in time, as we will do in this study. This will allow us to obtain new structural results and insights into periodic review inventory systems. Moreover, whereas models with a finite number of ordering opportunities typically have to be solved using time-consuming numerical techniques such as dynamic programming, our continuous formulation leads to simple newsvendor equations that determine the optimal ordering strategy during a review period. Interestingly, this strategy is also of a quite different nature than those proposed and studied before: it typically does not order at review moments.

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As we are the first to explore this problem, we will assume a negligible fixed ordering cost. This allows us to study the maximum benefit of continuous over periodic ordering, and also to obtain insightful analytical results. We do so under quite general conditions of a non-negative lead time and a general demand process. Although the main focus will be on continuous processes, we will also discuss the equivalent analysis for discrete demand distributions.

In line with previous periodic review studies (including those discussed above), we assume that no (partial) inventory updates are done between reviews. Obviously, this is relevant for situations where substantial effort is required to receive such updates. Despite the current technological improvements that facilitate and automate stock counting, the assumption that inventories can be completely checked on a continuous base is often unrealistic. Raman, DeHoratius, and Ton (2001) found evidence of inventory counting inaccuracy and product misplacement, Yano and Lee (1995) studied product quality issues, Nahmias (1982) analyzed spoilage due to product perishability, and Fleisch and Tellkamp (2005) performed a simulation study in which it was found that theft has severe consequences for the optimality of inventory policies that ignore resulting inaccuracies. Nevertheless, it is still worthwhile for future research to analyze whether partial information can be used to further lower costs, compared to not using that information at all, as is assumed in our initial exploration and more generally in the periodic review literature. We will return to this issue in the concluding section.

So, in our model, orders can be placed continuously and the quantity of interest is the order-up-to level for the inventory position at each time instant. We will derive the optimal policy in two phases. In the first phase, we assume that there is only one period and there is no salvage cost for inventory remaining at the end of the period. Any remaining inventory can be discarded free of charge. Given this simplifying assumption we formulate the total cost function and minimize it with respect to the order-up-to level at each time instant. The resulting policy will serve as the base-line for phase 2, where we consider the more realistic case with multiple periods in which remaining stock from any period remains present in the next period. We show that the optimal policy during a review period is to (i) not order until excess buffer stock remaining from the previous period is depleted, (ii) then apply continuous ordering following the base-line path for some time, but (iii) stop towards the end of the period in order to limit the excess buffer for the upcoming period.

The remainder of this paper is structured as follows. In Section 2 we derive the one-period base-line policy, and thereafter in Section 3 we adjust this policy to the general multi-period setting. In Section 4 we provide numerical examples and compare the policy to the periodic review, periodic ordering system, and in Section 5 we summarize our findings, discuss insights, and give concluding remarks.

2. The one-period problem: base-line model

Consider a single review period of length $T > 0$, for which at time 0 a stock level of 0 is observed. Stock information is updated only once per review period, at the start. However, non-negative orders can be placed at any time $t \in [0, T)$ and arrive after lead time $L \geq 0$. Demand D_r over a period of length r follows a distribution characterized by the continuous cdf F_{D_r} and corresponding pdf f_{D_r}. Holding costs per unit per time unit are $h > 0$ and shortage costs per unit per time unit are $p \geq h$. Fixed ordering costs are 0, and we can freely dispose of remaining inventory. Please note that since information on demand (including theft, misplacement, etc.) is not made available between reviews, demand during a review period is not subtracted from the inventory position. That is, the inventory position at any time during a review period is defined as the starting inventory position plus all orders placed since the start of the current review period.

Any inventory strategy is characterized by the order-up-to level $\mathcal{O}_t$ $(0 \leq t < T)$ at any time t during a review period. Note that since demands during a review period are not subtracted from the inventory position, only strategies with non-decreasing order-up-to levels need to be considered. The aim is to find the values for $\mathcal{O}_t$ that minimize the expected cost per period. An expression for that cost is obtained based on the following observation that holds for any $t \in [0, T)$: the inventory level at time $t + L$ is equal to the inventory position at time t minus the demand in interval $(0, t + L)$. Please note that we need to subtract demands in the interval $(0, t + L)$ and not only in the interval $(t, t + L)$, different from the standard analysis of continuous review inventory systems (see e.g. Axsäter, 2006, p. 90), since our definition of the inventory position at time t does not subtract the unknown demand in period $(0, t)$.

This problem is a one-period problem in the sense that effects on inventory positions after time T (or inventory levels after time $T + L$) are not taken into account. We seek for an optimal order-up-to level for any point in time in the interval $[0, T)$, so that total expected costs due to inventory levels in the interval $[L, T + L)$ are minimized. Any inventory that remains after time $T + L$ does not incur extra costs. Despite the restriction of the decision horizon to $[0, T)$, demand D_r is still defined for $r > T$, as is required in this model.

So, the total expected cost per cycle is

$$TC = \int_0^T [h\mathrm{E}(\mathcal{O}_t - D_{t+L})^+ + p\mathrm{E}(\mathcal{O}_t - D_{t+L})^-]\,\mathrm{d}t,$$

where $(x)^+ = \max\{0, x\}$ and $(x)^- = \max\{0, -x\}$. Obviously, no best solution for the whole period can be better than applying the optimal solution at any point during the period. Next, we therefore derive the optimal solution for a specific point in time during the period, after which we show that the point-for-point optimal solution indeed determines a feasible solution for the whole period as well.

For a specific value of t, the best value of $\mathcal{O}_t$ is the one that minimizes

$$\min_{\mathcal{O}_t} \{h\mathrm{E}(\mathcal{O}_t - D_{t+L})^+ + p\mathrm{E}(\mathcal{O}_t - D_{t+L})^-\}\,. \tag{1}$$

Using integration by parts, we easily get

$$\begin{aligned}
\mathrm{E}(\mathcal{O}_t - D_{t+L})^+ &= \int_{-\infty}^{\mathcal{O}_t} (\mathcal{O}_t - x)\,\mathrm{d}F_{D_{t+L}}(x) \\
&= \int_{-\infty}^{\mathcal{O}_t} \mathcal{O}_t\,\mathrm{d}F_{D_{t+L}}(x) - \int_{-\infty}^{\mathcal{O}_t} x\,\mathrm{d}F_{D_{t+L}}(x) \\
&= \mathcal{O}_t F_{D_{t+L}}(\mathcal{O}_t) - \mathcal{O}_t F_{D_{t+L}}(\mathcal{O}_t) + \int_{-\infty}^{\mathcal{O}_t} F_{D_{t+L}}(x)\,\mathrm{d}x \\
&= \int_{-\infty}^{\mathcal{O}_t} F_{D_{t+L}}(x)\,\mathrm{d}x,
\end{aligned}$$

and similarly

$$\mathrm{E}(\mathcal{O}_t - D_{t+L})^- = \int_{\mathcal{O}_t}^{\infty} [1 - F_{D_{t+L}}(x)]\,\mathrm{d}x.$$

It follows that the first order condition for (1) is

$$hF_{D_{t+L}}(\mathcal{O}_t) - p[1 - F_{D_{t+L}}(\mathcal{O}_t)] = 0.$$

As

$$\frac{\mathrm{d}^2 TC}{\mathrm{d}\mathcal{O}_t^2} = (p + h)f_{D_{t+L}}(\mathcal{O}_t) > 0,$$

TC is convex, and hence the found solution is indeed a minimum. So, the optimal order-up-to level $\tilde{\mathcal{O}}_t$ for a specific time $t \in [0, T)$, must satisfy