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Decision Support

A multi-step goal programming approach for group decision making with incomplete interval additive reciprocal comparison matrices

Zhou-Jing Wang[a,*], Kevin W. Li[b]

[a] School of Information, Zhejiang University of Finance & Economics, Hangzhou, Zhejiang 310018, China
[b] Odette School of Business, University of Windsor, Windsor, Ontario N9B 3P4, Canada

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ABSTRACT

This article presents a goal programming framework to solve group decision making problems where decision-makers' judgments are provided as incomplete interval additive reciprocal comparison matrices (IARCMs). New properties of multiplicative consistent IARCMs are put forward and used to define consistent incomplete IARCMs. A two-step goal programming method is developed to estimate missing values for an incomplete IARCM. The first step minimizes the inconsistency of the completed IARCMs and controls uncertainty ratios of the estimated judgments within an acceptable threshold, and the second step finds the most appropriate estimated missing values among the optimal solutions obtained from the previous step. A weighted geometric mean approach is proposed to aggregate individual IARCMs into a group IARCM by employing the lower bounds of the interval additive reciprocal judgments. A two-step procedure consisting of two goal programming models is established to derive interval weights from the group IARCM. The first model is devised to minimize the absolute difference between the logarithm of the group preference and that of the constructed multiplicative consistent judgment. The second model is developed to generate an interval-valued priority vector by maximizing the uncertainty ratio of the constructed consistent IARCM and incorporating the optimal objective value of the first model as a constraint. Two numerical examples are furnished to demonstrate validity and applicability of the proposed approach.

1. Introduction

The pairwise comparison method and hierarchy analysis technology have been widely used to decompose a complex multi-criteria decision making (MCDM) into a series of more tractable and simpler sub-problems. In a conventional analytic hierarchy process (AHP) (Saaty, 1980), a decision problem is structured as a hierarchy of criteria, sub-criteria and alternatives, and a multiplicative reciprocal comparison matrix is employed to express a decision-maker's pairwise comparison results, where the judgments are provided as crisp values. However, in many real-life decision problems, a decision-maker's judgments may contain vagueness and uncertainty and, hence, cannot be represented as crisp data (Dubois & Prade, 2012; Durbach & Stewart, 2012; Entani & Sugihara, 2012; Guo & Tanaka, 2010; Saaty & Vargas, 1987; Wan & Li, 2013; Xia & Chen, 2015; Xu & Chen, 2008; Zhu & Xu, 2014). As such, other forms of pairwise comparison matrices have been developed to deal with imprecise and uncertain judgment information, such as interval multiplicative reciprocal comparison matrices (Saaty & Vargas, 1987) and interval additive reciprocal comparison matrices (IARCM) (also called interval fuzzy preference relations (Xu & Chen, 2008)).

In a complete $n \times n$ comparison matrix, all judgment values are known. Given the reciprocity of a comparison matrix, it implies that the decision-maker should provide either the upper or lower diagonal $n(n-1)/2$ elements on a level with n alternatives or criteria. In reality, the decision-maker is sometimes unable or unwilling to provide his/her opinions over some alternatives due to insufficient information or limited expertise, especially in face of a large number of criteria or alternatives. In this situation, an incomplete comparison matrix is resulted (Alonso, Chiclana, Herrera, Herrera-Viedma, Alcalá-Fdez, & Porcel, 2008; Alonso, Herrera-Viedma, Chiclana, & Herrera, 2010; Chiclana, Herrera-Viedma, & Alonso, 2009; Chiclana, Herrera-Viedma, Alonso, & Herrera, 2008; Fedrizzi & Giove, 2007; Gong, 2008; Herrera-Viedma, Alonso, Chiclana, & Herrera, 2007; Liu, Zhang, & Wang, 2012; Liu, Pan, Xu, & Yu, 2012; Xu, 2004, 2012; Xu, Li, & Wang, 2014). MCDM with incomplete comparison matrices have been receiving increasing attention and many different methods have been developed to estimate missing or unknown values for incomplete additive reciprocal comparison matrices (Alonso et al., 2008, 2010;

* Corresponding author. Tel.: +86 571 85043562.
E-mail address: wangzj@xmu.edu.cn (Z.-J. Wang).

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Chiclana, Herrera-Viedma, & Alonso, 2009; Gong, 2008; Herrera-Viedma et al., 2007; Liu et al., 2012; Xu, 2004). For instance, Xu (2004) introduced the concept of incomplete additive reciprocal comparison matrices (or referred to as incomplete fuzzy preference relations), and proposed two goal programming models for obtaining priority weights of incomplete additive reciprocal comparison matrices from the viewpoints of additive transitivity and multiplicative consistency, respectively. An iterative procedure for estimating missing values was put forward by Herrera-Viedma et al. (2007) and applied to handle group decision making (GDM) problems with incomplete additive reciprocal comparison matrices based on additive transitivity. Liu et al., (2012) put forward a completion method by establishing a least squares model. Based on multiplicative consistency, Alonso et al. (2010) furnished a procedure to estimate missing values and developed a web-based consensus support system for GDM with incomplete additive reciprocal comparison matrices.

Genç, Boran, Akay, and Xu (2010) employed the feasible-region-based multiplicative transitivity (Xu & Chen, 2008) to develop two estimation approaches for incomplete IARCMs. Xia and Xu (2011) extended the functional equation proposed by Chiclana, Herrera-Viedma, Alonso, and Herrera (2009) to define perfect multiplicative consistent IARCMs and calculate missing values for incomplete IARCMs. From a multiplicative perspective, an interval additive reciprocal judgment can be transformed to an equivalent interval multiplicative reciprocal judgment (Liu, Zhang, & Zhang, 2014). After the conversion, the uncertainty level of the interval additive reciprocal judgment can be measured by the quotient of the upper and lower bounds of the corresponding interval multiplicative reciprocal judgment. Under this notion, a quotient of 1 indicates a crisp judgment without any uncertainty and the larger the ratio, the more uncertain the interval judgment. For the foresaid estimation methods in Genç et al. (2010) and Xia and Xu (2011), no mechanism is designed to consider the acceptability of the uncertainty levels of the estimated interval additive reciprocal judgments. As such, they sometimes yield highly uncertain estimated values. To obtain rational and reliable decision results, it is crucial to adapt the acceptable uncertainty levels of the estimated values as highly uncertain data contain little useful decision information.

In a GDM process, once all individual incomplete comparison matrices are completed and a group comparison matrix is obtained from the completed individual comparison matrices, a critical remaining issue is to derive a priority vector from the group comparison matrix. According to additive or multiplicative transitivity, different prioritization methods have been developed for obtaining an interval-valued priority vector from a complete interval reciprocal comparison matrices such as linear programs (Arbel, 1989; Guo & Wang, 2012; Hu, Ren, Lan, Wang, & Zheng, 2014; Kress, 1991; Wang, Lan, Ren, & Luo, 2012; Xu & Chen, 2008), nonlinear programs (Xia & Xu, 2014), and goal programs (Wang & Elhag, 2007; Wang & Li, 2012; Wang, Yang, & Xu, 2005).

Current research reveals that consistency properties are fundamental bases for estimating missing values and generating priority weights for pairwise comparison matrices. When decision-makers' pairwise comparisons are represented as incomplete IARCMs in a GDM problem, it is important to evaluate missing values first before a group priority vector is derived. Based on the multiplicative consistency concept proposed by Wang and Li (2012), new properties of consistent IARCMs are presented and employed to define multiplicative consistent incomplete IARCMs. A two-step framework consisting of two goal programs is developed to estimate missing values for incomplete IARCMs. The first step aims to estimate missing values such that the resulting complete IARCM possesses either multiplicative consistency or minimal inconsistency, and uncertainty ratios of the estimated values are controlled to be within an acceptable threshold specified by the decision-maker. This is accomplished by minimizing the absolute difference between the two sides of the logarithmic expression of the multiplicative transitivity equation and imposing acceptable uncertainty ratio constraints. The second step is established to find the most appropriate estimated missing values among the optimal solutions obtained from the first model. The modeling idea is that the missing values in an incomplete IARCM reflect the decision-maker's uncertainty about the pairwise comparison. Therefore, by incorporating the optimal solutions in the first model into its constraints, the second model maximizes the uncertainty ratio for the estimated interval additive reciprocal judgments to retain the decision-maker's inherent uncertainty in the original missing values. Subsequently, a weighted geometric mean approach is put forward to aggregate individual preferences into a group IARCM by directly employing the lower bounds of the interval additive reciprocal judgments (upper bounds are indirectly utilized due to reciprocity). It is shown that the group IARCM has multiplicative consistency if all individual IARCMs have multiplicative consistency. Next, a two-step procedure comprising two goal programs is established to derive interval weights from the aggregated group IARCM. By employing a parameterized transformation relation between multiplicative consistent IARCMs and interval weights, the first model minimizes the absolute difference between the logarithm of the group preference and that of the transformed consistent judgment such that the constructed multiplicative consistent IARCMs are the closest to the group IARCM. The second model determines the most appropriate interval-valued priority vector by maximizing the uncertainty ratio of the constructed consistent IARCM and employing the optimal objective value of the first model as a constraint. The optimal interval-valued priority vector derived from the second model is able to be transformed to an IARCM with multiplicative consistency that is closest to that obtained by interval arithmetic and the group IARCM. Finally, by putting the aforesaid models together, an algorithm is proposed for solving GDM problems with incomplete IARCMs.

The remainder of the paper is organized as follows. Section 2 reviews some basic concepts related to additive reciprocal comparison matrices and IARCMs. New properties of multiplicative consistent IARCMs and the multiplicative consistency definition of incomplete IARCMs are introduced in Section 3. Section 4 develops two goal programs for estimating missing values in an incomplete IARCM. A goal programming approach is presented for generating an interval-valued priority vector of the group IARCM and a procedure is further put forward to solve GDM problems with incomplete IARCMs in Section 5. Section 6 provides concluding remarks.

2. Preliminaries

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of n alternatives, if a pairwise comparison matrix $R = (r_{ij})_{n\times n}$ on X satisfies

$$r_{ij} \in [0, 1], \quad r_{ij} + r_{ji} = 1, \quad r_{ii} = 0.5, \qquad \forall i, j = 1, 2, \ldots, n, \tag{2.1}$$

then $R = (r_{ij})_{n\times n}$ is called an additive reciprocal comparison matrix (or referred to as an additive reciprocal preference relation (De Baets & De Meyer, 2005; De Baets, De Meyer, & De Loof, 2010)).

Element r_{ij} in R denotes the $[0, 1]$-valued preference or importance degree of x_i over x_j. The larger the value of r_{ij}, the smaller the value of $r_{ji} = 1 - r_{ij}$ and the stronger the preference ratio $\frac{r_{ij}}{r_{ji}}$ of x_i over x_j. $r_{ij} > 0.5$ indicates that $\frac{r_{ij}}{r_{ji}} > 1$ and x_i is superior to x_j with the preference ratio $\frac{r_{ij}}{r_{ji}}$. $r_{ij} < 0.5$ shows that $\frac{r_{ij}}{r_{ji}} < 1$ and x_i is non-preferred to x_j with the preference ratio $\frac{r_{ij}}{r_{ji}}$. Especially, if $r_{ij} = 0.5$, then $\frac{r_{ij}}{r_{ji}} = 1$, implying that x_i and x_j are equally preferred.

Definition 2.1. (Tanino, 1984) Let $R = (r_{ij})_{n\times n}$ be an additive reciprocal comparison matrix with $0 < r_{ij} < 1, \forall i, j = 1, 2, \ldots, n$. If R satisfies the following transitivity condition: