European Journal of Operational Research 242 (2015) 933–941

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

A multi-period fuzzy portfolio optimization model with minimum transaction lots

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ARTICLE INFO

Article history:
Received 30 December 2013
Accepted 28 October 2014
Available online 18 November 2014

Keywords:
Finance
Multi-period portfolio selection
Mean-semivariance
Minimum transaction lots
Genetic algorithm

ABSTRACT

In this paper, we consider a multi-period fuzzy portfolio optimization problem with minimum transaction lots. Based on possibility theory, we formulate a mean-semivariance portfolio selection model with the objectives of maximizing the terminal wealth and minimizing the cumulative risk over the whole investment horizon. In the proposed model, we take the return, risk, transaction costs, diversification degree, cardinality constraint and minimum transaction lots into consideration. To reflect investor's aspiration levels for the two objectives, a fuzzy decision technique is employed to transform the proposed model into a single objective mixed-integer nonlinear programming problem. Then, we design a genetic algorithm for solution. Finally, we give an empirical application in Chinese stock markets to demonstrate the idea of our model and the effectiveness of the designed algorithm.

1. Introduction

Markowitz (1952) originally proposed mean-variance model for portfolio selection, which laid the foundation of modern portfolio analysis. Portfolio optimization deals with the problem of how to allocate investor's wealth among a basket of securities. To realize this idea, the proper portfolio model must be presented. Most of existing portfolio selection models have been proposed on the assumption of a perfect fractionability of the investments, which are difficult to implement. In real world, each security has its minimum transaction lot. So it is necessary to consider rounds. To reflect this realistic characteristic of security, some researchers have proposed a series of mixed-integer programming models. Speranza (1996) proposed a mixed-integer programming model on the basis of the mean absolute deviation model in Konno and Yamazaki (1991) by taking minimum transaction lots and maximum number of securities into consideration, and designed a simple two-phase heuristic algorithm to solve the proposed model. Mansini and Speranza (1999) considered a NP-complete portfolio selection problem with minimum lots and presented three heuristic algorithms to solve the problem. In Kellerer, Mansini, and Speranza (2000), a portfolio selection model with fixed costs and minimum transaction lots was proposed and two linear programming based heuristic algorithms were designed for solution. Konno and Wijayanayake (2001) discussed portfolio optimization problem with concave transaction costs and minimum transaction lots under the framework of mean-absolute deviation risk measure, and devised a branch and bound algorithm for solving the proposed model. Konno and Wijayanayake (2002) discussed the market illiquidity effects and investigated a portfolio optimization problem with D. C. transaction costs and minimal transaction unit constraints. Recently, Lin and Liu (2008) proposed three possible models for portfolio selection problems with minimum transaction lots, and devised genetic algorithms to solve them. Baixauli-Soler, Alfaro-Cid, and Fernandez-Blanco (2011) employed VaR as risk measure to investigate an asset allocation problem under real constraints, such as minimum transaction lots and non-linear cost structure.

Notice that all the models mentioned above are single period portfolio selection models. However, in real world, investors tend to invest long-term investment. The investors should adjust their wealth from time to time. So it is nature to investigate multi-period portfolio selection problems. Numerous researchers have studied multi-period portfolio optimization problems, see for instance, Elton and Gruber (1974), Fama (1970) and Hakansson (1971). Li and Ng (2000) made breakthrough result for dynamic portfolio. In their model, they employed the idea of embedding the problem in a tractable auxiliary problem to investigate the mean-variance formulation in multi-period portfolio selection and obtained the corresponding mean-variance efficient frontier. After that, Zhu, Li, and Wang (2004) used the same approach to study the continuous-time dynamic multi-period problem by taking risk control into account. Rocha and Kuhn (2012) formulated a dynamic mean-variance model for electricity portfolio management. Gülpınar and Rustem (2007) presented min–max

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http://dx.doi.org/10.1016/j.ejor.2014.10.061
0377-2217/

formulations of multi-period mean-variance optimization problem with multiple rival risk and return scenarios. Fonseca and Rustem (2012) applied linear decision rules to a robust multiperiod international portfolio. Costa and Araujo (2008) considered a multi-period generalized mean-variance model with Markov switching in the key market parameters. Calafiore (2008) concerned with multi-period sequential decision problems for financial asset allocation and presented a multi-period portfolio optimization model with control policies. Briec and Kerstens (2009) proposed a general approach for multi-horizon mean-variance portfolio analysis. Fu, Lari-Lavassani, and Li (2010) considered continuous-time mean-variance portfolio selection with borrowing constraint, i.e., under different interest rates for borrowing and lending, rendering the market incomplete. Fu, Wei, and Yang (2014) considered the optimal asset allocation problem in a continuous-time regime-switching market. Çelikyurt and Özekici (2007) presented several multi-period portfolio optimization models where the market consists of a riskless asset and several risky assets. The literatures mentioned were proposed on the framework of probability theory.

In probabilistic portfolio models, the returns of risky assets are regarded as random variables with probability distributions. The basic assumption of them is that the future situation of risky assets can be correctly reflected by its historical data. However, since financial markets are complex and ever-changing, this kind of assumption is hard to ensure in real world. As is well known, portfolio decision-making is often affected by many non-probabilistic factors including social, economic, political, people's cognitive and psychological factors, etc. For example, REITs and the financial crisis mentioned by Basse, Friedrich, and Vazquez Bea (2009) are also affected by aforementioned human factors, which have caused some instability in markets affecting the correlations between the returns of different asset classes and that this observation is a reminder of the fact that the correlation matrices of returns on risky assets regularly used in financial optimizations are not necessarily stable over time. Thus, the influence of experts' experiences and knowledge, and investors' subjective opinions on portfolio selection cannot be neglected (such as Tanaka & Guo, 1999, Fang, Lai, & Wang, 2006, Vercher, Bermúdez, & Segura, 2007, Liu, Zhang, & Xu, 2012 and so on). Due to the influence of above-mentioned non-probabilistic factors, the future situation of risky assets is usually characterized by fuzzy uncertainty such as vagueness and ambiguity. Namely, decision makers are usually provided with information which is characterized by vague linguistic descriptions such as high risk, low profit, high interest rate, etc. Notice that all these factors mentioned above are affected by human's subjective opinions. They have a great influence on financial markets such that the returns of risky assets are characterized by vague linguistic values or fuzzy values in many cases. Taking human's subjective factors into consideration, fuzzy approaches are more suitable than probabilistic approaches in characterizing the uncertainty in real financial markets as pointed by Liu and Zhang (2013). With the wide use of fuzzy set theory in Zadeh (1965), more and more researchers have realized that they could use the fuzzy set theory to handle the vagueness and ambiguity, see for example, Tanaka and Guo (1999), Carlsson, Fullér, and Majlender (2002), Fang et al. (2006), Vercher et al. (2007), Zhang, Wang, Chen, and Nie (2007), Zhang, Zhang, and Xiao (2009), Zhang et al. (2010), Barak, Abessi, and Modarres (2013), and Liu and Zhang (2013). Though great progress has been made in fuzzy portfolio selection, most of existing models are single period models. Recently, Sadjadi, Seyedhosseini, and Hassanlou (2011) discussed a fuzzy multi-period portfolio optimization problem with different rates for borrowing and lending. Zhang, Liu, and Xu (2012) proposed a possibilistic mean-semivariance-entropy model for multi-period fuzzy portfolio selection. Liu et al. (2012) presented four multi-period fuzzy portfolio optimization models by using multiple criteria. To our knowledge, the researches about multi-period fuzzy portfolio selection problem with real constrains are few. The purpose of this paper is to investigate multi-period portfolio selection problem in fuzzy environment with minimum transaction lots. We present a mean-semivariance model by using possibility theory. In the proposed model, we consider six criteria, including the return, risk, transaction cost, diversification degree, cardinality constraint and minimum transaction lots. The investment return is measured by possibilistic mean value of the return rate of a portfolio. The investment risk is quantified by the lower possibilistic semivariance of the return rate of portfolio and the diversification degree is measured by the proportion entropy in Kapur (1990). Since the proposed model is a bi-objective programming problem, an S-shape membership function is used to express investor's satisfaction degree for each objective and then transform it into a corresponding single objective mixed-integer nonlinear programming problem. After that, we design a novel genetic algorithm for solution.

The rest of the paper is organized as follows. In Section 2, we introduce some basic conceptions about fuzzy variables. In Section 3, we first formulate a bi-objective multi-period portfolio optimization model with minimum lots, in which we assume that the investor wants to maximize the terminal wealth and minimize the cumulative risk over all investment horizon. Then, we employ a nonlinear S-shape fuzzy membership functions to express investor's aspiration levels for the two objectives. By using the nonlinear S-shape membership functions, we transform the proposed model into a single programming problem. In Section 4, we design a genetic algorithm to solve the proposed model. In Section 5, we use an empirical application in Chinese stock markets to demonstrate the idea of our model and the effectiveness of the designed algorithm for solution. In Section 6, we conclude this paper by some remarks.

2. Preliminaries

In this section, let us first review some basic conceptions about fuzzy variables, which we need in the following sections.

Let A be a fuzzy number, i.e. such fuzzy subset A of the real line $\mathbb{R}$ with a membership function $\mu_A : \mathbb{R} \longrightarrow [0, 1]$, that (see Dubois & Prade, 1980):

(I) A is normal, i.e. there exists an element x_0 such that $\mu_A(x_0) = 1$;
(II) A is fuzzy convex, i.e. $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$, $(\forall x_1, x_2 \in \mathbb{R}; \lambda \in [0, 1])$;
(III) μ_A is upper semicontinuous;
(IV) $supp(A)$ is bounded, where $supp(A) = cl\{x \in \mathbb{R} | \mu_A(x) > \gamma\}$ and cl is the γ-level set of A as $[A]^\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$ $(\forall \gamma \in [0, 1])$ closure operator.

Denote the family of fuzzy numbers as $\mathcal{F}$. For any $A \in \mathcal{F}$, we denote the γ-level set of A as $[A]^\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$ $(\gamma \in [0, 1])$. Fuzzy numbers can also be considered as possibility distributions. If $A \in \mathcal{F}$ is a fuzzy number and $x \in \mathbb{R}$ a real number then $\mu_A(x)$ can be interpreted as the degree of possibility of the statement 'x is A'.

Let $A \in \mathcal{F}$ be a fuzzy number with $[A]^\gamma = [\underline{a}(\gamma), \overline{a}(\gamma)]$ $(\gamma \in [0, 1])$. Carlsson and Fullér (2001) defined the possibilistic mean value of fuzzy number A as follows

$$E(A) = \int_0^1 \gamma(\underline{a}(\gamma) + \overline{a}(\gamma))\mathrm{d}\gamma. \tag{1}$$

In Saeidifar and Pasha (2009), if we set the weighted function as $f(\gamma) = 2\gamma$, then we have the lower and upper possibilistic variances of A with the following forms

$$\mathrm{Var}^-(A) = 2\int_0^1 \gamma(E(A) - \underline{a}(\gamma))^2\mathrm{d}\gamma, \tag{2}$$

$$\mathrm{Var}^+(A) = 2\int_0^1 \gamma[E(A) - \underline{a}(\gamma)]^2\mathrm{d}\gamma. \tag{3}$$

In particular, if A is a symmetrical fuzzy number, then $\mathrm{Var}^-(A) = \mathrm{Var}^+(A)$.