European Journal of Operational Research 242 (2015) 1008–1016

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Mathematical formulations for a 1-full-truckload pickup-and-delivery problem

Michel Gendreau[a], Jenny Nossack[b], Erwin Pesch[b,*]

[a] CIRRELT and MAGI, École Polytechnique de Montréal, C.P. 6079, succ. Centre-ville, Montréal, Québec, Canada H3C 3A7, Canada
[b] Department of Management Information Science, Universität Siegen, Hölderlinstraße 3, 57068 Siegen, Germany

ARTICLE INFO

Article history:
Received 11 September 2012
Accepted 24 October 2014
Available online 3 November 2014

Keywords:
Pickup and delivery problem
Asymmetric traveling salesman problem
Classical Benders decomposition
Generalized Benders decomposition

ABSTRACT

We address a generalization of the asymmetric Traveling Salesman Problem where routes have to be constructed to satisfy customer requests, which either involve the pickup or delivery of a single commodity. A vehicle is to be routed such that the demand and the supply of the customers is satisfied under the objective to minimize the total distance traveled. The commodities which are collected from the pickup customers can be used to accommodate the demand of the delivery customers. In this paper, we present three mathematical formulations for this problem class and apply branch-and-cut algorithms to optimally solve the model formulations. For two of the models we derive Benders cuts based on the classical and the generalized Benders decomposition. Finally, we analyze the different mathematical formulations and associated solution approaches on well-known data sets from the literature.

1. Introduction

The problem studied in this paper is a generalization of the asymmetric Traveling Salesman Problem (TSP) in which the set of customers is divided into pickup and delivery customers and where the former supplies and the latter demands one unit of a single commodity. A vehicle is to be routed such that the supply and the demand of the customers is satisfied while minimizing the total distance traveled. We refer to this routing problem as one-commodity Full-Truckload Pickup-and-Delivery Problem (1-FTPDP). The term full-truckload implies unit capacity and unit supply/demand of the vehicle and the customers, respectively (Parragh, Doerner, & Hartl, 2008b). The 1-FTPDP belongs to the class of many-to-many Pickup and Delivery Problems (PDP) where each unit of a pickup customer can be used to accommodate the demand of any delivery customer (Berbeglia, Cordeau, Gribkovskaia, & Laporte, 2007). Besides the exchange of commodities between customers, the depot has the capacity to fulfill the customers' supply and demand. For the sake of simplicity, we assume that the depot has a sufficient number of commodity available and enough space for commodity storage. Note that this is a general assumption in the literature (refer, e.g., to Hernández-Pérez & Salazar-González, 2004a; Martinovic, Aleksi, & Baumgartner, 2008).

A real-life application of the 1-FTPDP arises, for example, in the pre- and end-haulage of intermodal container transportation. Intermodal container transportation denotes the movement of containers by two or more transportation modes (rail, maritime, and road) in a single transport chain, where the change of modes is performed at bi- and tri-modal terminals (Macharis & Bontekoning, 2004). The route of intermodal transport is namely subdivided into the pre-, main-, and end-haulage, denoting the route segments from customer to terminal, terminal to terminal, and terminal to customer, respectively. The main-haulage generally implies the longest traveling distance and is typically carried out by rail or maritime, whereas the pre- and end-haulage are handled by trucks to enable door-to-door transports. The transportation assignments that arise in the pre- and end-haulage are the movements of fully-loaded containers from customers to terminals and vice versa. In addition, empty containers are considered as transportation resources and are provided by the carrier for freight transportation. The carrier's objective is to sequence the fully-loaded container transportations such that the total traveling cost is minimized. Furthermore, it is part of the optimization to decide where to deliver the empty containers released at the receiver customers and where to pickup the empty containers for the shipper customers. This outlined routing problem can be modeled as a 1-FTPDP, where each receiver customer is regarded as (empty container) pickup customer and each shipper customer as (empty container) delivery customer. For further details on the real-life application, we refer the reader to Zhang, Yun, and Kopfer (2010) and Nossack and Pesch (2013).

* Corresponding author: Tel.: 0049 271 7402420.
E-mail address: erwin.pesch@uni-siegen.de (E. Pesch).

http://dx.doi.org/10.1016/j.ejor.2014.10.053

The literature on PDPs is quite extensive. Savelsbergh and Sol (1995), Berbeglia et al. (2007), Parragh, Doerner, and Hartl (2008a), Parragh et al. (2008b), Pillac, Gendreau, Guéret, and Medaglia (2013), and Lahyani, Khemakhem, and Semet (2015) provide detailed surveys of the recent literature, as well as classification schemes. We follow Berbeglia et al. (2007) by differentiating between many-to-many, one-to-many-to-one, and one-to-one PDPs. The most frequently encountered PDPs are the ones with a one-to-one structure, where each commodity has a defined pickup and delivery location. Problems of this type arise, for example, in courier and door-to-door transportation (refer, e.g., to Cordeau & Laporte, 2003). In problems with a one-to-many-to-one relationship, commodities are initially located at the depots and are delivered to the delivery customers, whereas the commodities that are picked up at the pickup customers are destined to the depots. Real-world applications arise, for example, in the delivery of beverages and the collection of empty bottles (refer, e.g., to Gendreau, Laporte, & Vigo, 1999). The 1-FTPDP belongs to the class of PDPs with a many-to-many dependency where the supply of any pickup customer can be accommodated by any other delivery customer.

The literature on PDPs with a many-to-many relationship is rather limited and mainly focuses on the single vehicle case. This problem is denoted in the literature as Pickup-and-Delivery Traveling Salesman Problem (PDTSP). If the PDTSP is restricted explicitly to a single commodity, it is referred to as 1-PDTSP. Chalasani and Motwani (1999) address a special case of the 1-PDTSP by considering unit supply/demand of the customers and finite vehicle capacity. The authors call this problem Q-delivery TSP (Q denotes the vehicle capacity). They propose a 9.5-approximation algorithm for $Q \in \mathbb{R}^+$ and a 2-approximation algorithm for $Q = 1$ and $Q = \infty$. Anily and Bramel (1999) present a $(7 - 3/Q)$-approximation algorithm for the same problem with $Q \in \mathbb{R}^+$ and refer to it as Capacitated Traveling Salesman Problem with Pickups and Deliveries. Hernández-Pérez and Salazar-González (2004a) develop a branch-and-cut algorithm using Benders decomposition to optimally solve instances of the 1-PDTSP. The authors consider real-valued supply/demand and finite vehicle capacity. Wang and Lim (2006) propose polynomial time algorithms for the same problem with unit supply/demand on a path and a tree graph topology. Hernández-Pérez and Salazar-González (2004b) suggest two heuristics for the 1-PDTSP with real-valued supply/demand and finite vehicle capacity. One heuristic is based on a nearest neighbor and a 2-opt/3-opt approach and the other applies the branch-and-cut algorithm presented in Hernández-Pérez and Salazar-González (2004a) on restricted feasible sets. Moreover, Martinovic et al. (2008) solve instances of the 1-PDTSP by an iterative modified simulated annealing algorithm, Hernández-Pérez, Rodríguez-Martín, and Salazar-González (2009) by a hybrid GRASP/VND heuristic, Zhao, Li, Sun, and Mei (2009) by a genetic algorithm, and Hosny and Mumford (2010) by a VNS/SA approach. The PDTSP with multiple commodities, unit supply/demand, and unit vehicle capacity has been addressed by Anily and Hassin (1992). The authors propose a 2.5-approximation algorithm for this so-called swapping problem. Furthermore, Anily, Gendreau, and Laporte (1999) address the swapping problem on a line and propose an exact, polynomial time algorithm.

The considered 1-FTPDP is NP-hard. To verify its computational complexity, we refer the reader to Anily and Hassin (1992). They prove the NP-hardness of the swapping problem by showing that even the simplest problem (namely the 1-FTPDP) is NP-hard.

The key contribution of our work is to present various mathematical formulations for the 1-FTPDP and to analyze their performances in a computational study. The nature of the 1-FTPDP points to decomposition methods in which the problem is partitioned into a routing and an assignment problem. We propose two so-called integrated formulations that are suited for decomposition and which capture the routing and the assignment structure of the 1-FTPDP. We apply the classical and the generalized Benders decomposition (Benders, 1962; Geoffrion, 1972) to these integrated formulations and study their computational performances. Furthermore, we compare the results to a classical asymmetric TSP formulation.

The remainder of the paper is organized as follows. A detailed description of the various mathematical formulations are given in Section 2. Branch-and-cut solution algorithms for the different mathematical models are described in Section 3. In Section 4, we summarize the results of our computational study which we conducted on several instances to assess the computational performance of the algorithms. Finally, we conclude our research in Section 5.

2. Mathematical formulations for the 1-FTPDP

In the following, we will present different model formulations for the 1-FTPDP. The following notation is used throughout the paper. Let 0 denote the depot, $C^P = \{1, \ldots, n_1\}$ the set of pickup customers, and $C^D = \{n_1 + 1, \ldots, n_2\}$ the set of delivery customers. Based on the property that the depot is assumed to provide and receive a sufficient amount of a given commodity, the depot may either be considered as pickup or as delivery customer. Hence, to ensure the supply/demand of the customers, we add an appropriate number of depot duplicates to the set of pickup/delivery customers.

2.1. Asymmetric TSP formulation

The 1-FTPDP can simply be solved as a classical asymmetric TSP (refer, e.g., to Dantzig, Fulkerson, & Johnson, 1954). The according model formulation is thereby defined on a digraph $G' = (V', A')$, where V' represents the vertex set and A' the set of directed edges. V' consists of the depot 0, the set of pickup customers C^P, and the set of delivery customers C^D. Directed edges $(i, j) \in A'$ are defined between any pair of vertices $i, j \in V'$ with $i \neq j$ and symbolize vehicle movements. The traveling distance between any two locations $(i, j) \in A'$ is denoted by the edge weight $c(i, j) \in \mathbb{R}^+$. Note that the traveling distance between two locations may be different, i.e. $c(i, j) \neq c(j, i)$, and edges that correspond to infeasible vehicle movements have edge weight ∞ and are referred to as infeasible edges. For instance, edges $(i, j) \in A'$ among pickup customers, i.e., $i, j \in C^P$, $i \neq j$, and among delivery customers, i.e., $i, j \in C^D$, $i \neq j$, are infeasible. We incorporate binary decision variables $y'_{ij} \in \{0, 1\}$ for each directed edge $(i, j) \in A'$ to denote whether ($y'_{ij} = 1$) or not ($y'_{ij} = 0$) edge (i, j) is traversed by the vehicle. The TSP formulation is further denoted by P^{TSP} and is given by the following model. Moreover, let $\mathbf{y}' := (y'_{ij} | i, j \in V', i \neq j)$.

$$\min \sum_{(i,j)\in A'} c(i,j) \cdot y'_{ij} \tag{2.1}$$

$$\text{s.t.} \sum_{\substack{j\in V'\\ j\neq i}} y'_{ji} = 1 \quad \forall i \in V' \tag{2.2}$$

$$\sum_{\substack{j\in V'\\ j\neq i}} y'_{ij} = 1 \quad \forall i \in V' \tag{2.3}$$

$$\sum_{\substack{i,j\in S\\ j\neq i}} y'_{ij} \leq |S| - 1 \quad \forall S \subset V' \tag{2.4}$$

$$y'_{ij} \in \{0, 1\} \quad \forall i, j \in V', i \neq j \tag{2.5}$$

Objective function (2.1) minimizes the total traveling distance. Constraints (2.2) and (2.3) ensure that each pickup customer and each delivery customer, as well as the depot is entered and left exactly once. Constraints (2.4) are the classical subtour elimination constraints that impose route connectivity. Finally, constraints (2.5) define the domains of the decision variables.