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Innovative Applications of O.R.

Solving the Aircraft Landing Problem with time discretization approach

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ABSTRACT

This paper studies the multiple runway Aircraft Landing Problem. The aim is to schedule arriving aircraft to available runways at the airport. Landing times lie within predefined time windows and safety separation constraints between two successive landings must be satisfied. We propose a new approach for solving the problem. The method is based on an approximation of the separation time matrix and on time discretization. The separation matrix is approximated by a rank two matrix. This provides lower bounds or upper bounds depending on the choice of the approximating matrix. These bounds are used in a constraint generation algorithm to, exactly or heuristically, solve the problem. Computational tests, performed on publicly available problems involving up to 500 aircraft, show the efficiency of the approach.

1. Introduction

This paper addresses the problem of scheduling aircraft landings at an airport. Given a set of planes, the problem is one of assigning a runway and a landing time for each plane. Each plane has to land within its predefined time window and safety separation distances have to be maintained between any pair of planes. This problem, referenced in the literature as the Aircraft Landing Problem (ALP), has been extensively studied. Beasley, Krishnamoorthy, Sharaiha, and Abramson (2000) present a mixed integer zero-one formulation of the problem. Each plane has a target landing time within its time window. A cost is accounted when a plane lands after or before its target time. The objective is to minimize the total cost of deviation from the target times. The problem is solved optimally with a linear programming based tree search algorithm. A heuristic method is also proposed. Using a similar model, Ernst, Krishnamoorthy, and Storer (1999) propose a different heuristic approach. When the binary variables are fixed the remaining continuous linear problem is solved by a specialized simplex method which evaluates the landing times very quickly. The method is used in a space search heuristic as well as a branch and bound algorithm. Fahle, Feldmann, Gotz, Grothklags, and Monien (2003) present a comparison of several models and heuristics. They study a MIP (mixed integer program) and an integer linear program using time discretization introduced in Beasley et al. (2000). They also compare two heuristic algorithms, Hill Climbing and Simulated Annealing methods. A SAT (Satisfiability Problem) formulation is proposed to decide if there exists a valid solution for the problem. In Hansen's (2004) paper, four genetic algorithms are tested for the problem. The author introduces runway dependent time windows. Computational results are given for small instances. Beasley, Sonander, and Havelock (2001) use a population heuristic to solve a problem instance based on observations during a busy period at London Heathrow airport. Diallo, Ndiaye, and Seck (2012) solve the problem for the single runway case. Experiments based on real datasets of Léopold Sédar Senghor Airport of Dakar are presented. Pinol and Beasley (2006) consider extensions of the previous model given in Beasley et al. (2000). Time windows and separation distance between two successive aircraft are assumed to be runway dependent. These new criteria involve respectively linear and quadratic constraints. Next, two population heuristics are applied to the problem. In order to consider airline preferences for individual flights, a new objective function is introduced by Soomer and Franx (2008). Equity considerations lead to a convex piecewise linear function which has to be minimized. The problem is solved by a local search heuristic. Artiouchine, Baptiste, and Durr (2008) consider the case where more than one time window is assigned to each plane. These time windows modelize the mechanisms as vector spaces and holding patterns, used by air controllers to delay planes. The separation criteria are not a constraint but the objective is to maximize the minimum elapsed time between any two consecutive landings. The authors study the complexity of the problem and identify polynomially solvable cases. For solving general problems, a branch and cut framework is used. Bianco, Dell'Olmo, and Giordani (2006) propose a job-shop scheduling model with sequence dependent processing times. Jobs modelize planes. The paths of the planes in the terminal area are decomposed into machines like runways. The processing times take into account the separation distances between planes. The model is quite general and is suitable for arrivals and departures. Several objectives are considered as average delay

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minimization, minimization of maximum delay and throughput (capacity) maximization. The problem is solved by a heuristic method based on a fast local search. Bencheikh, Boukachour, El Hilali Alaoui, and El Khoukhi (2009) modelize the landing problem as a job-shop scheduling problem. The problem is solved by a hybrid method combining genetic algorithms with an ant colony optimization algorithm. D'Ariano, Pistelli, and Pacciarelli (2012) consider take-off and landing operations at a busy terminal area. Aircraft timing and routing issue is modelized as a job-shop scheduling problem and is solved by a truncated branch and bound algorithm for fixed routes. Aircraft rerouting is performed by a tabu search in order to improve the solution. The overall algorithm is tested on practical instances from Rome Fiumicino airport. Diaz and Mena (2005) and van Leeuwen, Hesselink, and Rohling (2002) have used constraint programming. These methods are well suited for small instances. However, the quality of the provided solutions is not as good as for larger instances. The dynamic case of ALP is considered by Beasley, Krishnamoorthy, Sharaiha, and Abramson (2004). Decisions must be taken in a dynamic fashion as time passes. Each new decision must take into account the previous decision which was made. The problem is solved using two types of heuristics.

In this paper, we propose a new approach for solving the Aircraft Landing Problem. The method is based on an approximation of the separation time matrix and on time discretization. The separation time matrix is approximated by a rank two matrix.

In the ideal case, the separation time matrix is a rank two matrix. In this case, the problem is stated as a 0-1 linear program. The LP relaxation of this model is very tight. Discretization time induces an important number of variables but this drawback is counterbalanced by very good LP relaxation. The discretization methods proposed in Beasley et al. (2000) and Fahle et al. (2003) do not have this property.

When the separation time matrix is not a rank 2 matrix, an approximation is done. This provides lower bounds or upper bounds depending on the choice of the approximating matrix. These bounds are used in a constraint generation algorithm to, optimally or heuristically, solve the problem.

In Section 2, we recall the classical formulation of the problem. In Section 3, we show that the problem can be discretized. Next, in Section 4, we give the model based on time discretization and on a rank two separation time matrix. In Section 5, we present the approximation method of the separation time matrix, the exact algorithm and the heuristic method based on constraint generation. Computation results are reported in Section 6 and concluding remarks follow in Section 7.

2. Classical modelization

Each aircraft entering within the radar range at its destination airport, receives instructions from air traffic control. A landing time and a runway on which to land are assigned to the plane. The landing time must be between an earliest landing time and a latest landing time. The earliest landing time corresponds to the time at which the aircraft can land if it flies at its fastest speed. To delay the landing time, the speed of the aircraft can be decreased or the flight plan can be lengthened by circling. The latest time corresponds to the maximum landing time achievable by these delaying mechanisms. Within this time window, there is a target time which corresponds to the time at which aircraft can land if it flies at its cruise speed. The target time is the preferred landing time. Safety distances between pair of successive planes must be respected. Separation distances are converted into separation times using a fixed landing speed depending on the aircraft type. Then, there must be a minimal lapse of time between the landing of a plane and the landing of any successive plane. Separation time holds between a pair of planes landing on the same runway or on different runways.

We give a modelization of the Aircraft Landing Problem which is based on the one presented in Beasley et al. (2000) and Pinol and Beasley (2006).

The data are the following:

- P set of planes, R set of runways available for landing
- E_i the earliest landing time for plane i, $\forall i \in P$
- T_i the target (preferred) landing time for plane i, $\forall i \in P$
- L_i the latest landing time for plane i, $\forall i \in P$
- $S_{ij} \geq 0$ the minimum separation time between planes i and j where i lands before j on the same runway
- $s_{ij} \geq 0$ the minimum separation time between planes i and j where i lands before j on a different runway

S_{ij} is called the longitudinal separation time between leading aircraft i and trailing aircraft j. s_{ij} is the diagonal separation time (cf. Bianco et al., 2006). We assume $S_{ij} > s_{ij}$.

The decision variables are the following:

- δ^1_{ij} $\forall i, j \in P$ $(i \neq j)$ binary variable such that $\delta^1_{ij} = 1$ if and only if aircraft i lands before aircraft j and on the same runway.
- δ^2_{ij} $\forall i, j \in P$ $(i \neq j)$ binary variable such that $\delta^2_{ij} = 1$ if and only if aircraft i lands before aircraft j and on a different runway.
- z_{ij} $\forall i, j \in P$ $(i < j)$ binary variable such that $z_{ij} = 1$ if and only if aircraft i and j land on the same runway.
- y_{ir} $\forall i \in P, r \in R$ binary variable such that $y_{ir} = 1$ if and only if aircraft i lands on runway r.
- $x_i \geq 0$ the scheduled landing time for plane i, $\forall i \in P$.

The constraints are the following:

$$(P_B)\begin{cases} x_j \geq x_i + S_{ij} + (1-\delta^1_{ij})(-S_{ij} - L_i + E_j), \quad \forall i \neq j \in P & (1)\\ x_j \geq x_i + s_{ij} + (1-\delta^2_{ij})(-s_{ij} - L_i + E_j), \quad \forall i \neq j \in P & (2)\\ \delta^1_{ij} + \delta^1_{ji} = z_{ij}, \quad \forall i < j \in P & (3)\\ \delta^2_{ij} + \delta^2_{ji} = 1 - z_{ij}, \quad \forall i < j \in P & (4)\\ z_{ij} \geq y_{ir} + y_{jr} - 1, \quad \forall i < j \in P \quad \forall r \in R & (5)\\ \sum_{r \in R} y_{ir} = 1, \quad \forall i \in P & (6)\\ E_i \leq x_i \leq L_i, \quad \forall i \in P & (7)\\ \delta^1_{ij}, \delta^2_{ij} \in \{0,1\}, \quad \forall i \neq j \in P & \\ z_{ij} \in \{0,1\}, \quad \forall i < j \in P, y_{ir} \in \{0,1\}, \quad \forall i \in P \quad \forall r \in R & \end{cases}$$

We give a slightly different formulation of the one given in Beasley et al. (2000) and Pinol and Beasley (2006). Constraints (1) are related to planes landing on the same runway, while constraints (2) are related to planes landing on different runways. They ensure minimum separation time in each case if $\delta^1_{ij} = 1$ or $\delta^2_{ij} = 1$. If $\delta^1_{ij} = 0$ then (1) becomes $x_j \geq x_i - L_i + E_j$ which is true by (7). The same holds for (2). Hence for $\delta^1_{ij} = 0$ and $\delta^2_{ij} = 0$, these constraints are inactive. In Beasley et al. (2000) and Pinol and Beasley (2006), conditions (1), (2) are aggregated in a single set of constraints.

If planes i and j $(i < j)$ land on the same runway, i.e. $z_{ij} = 1$, constraint (3) becomes $\delta^1_{ij} + \delta^1_{ji} = 1$ meaning that either aircraft i or j must land first. If planes i and j land on different runways, i.e. $z_{ij} = 0$, then $\delta^1_{ij} + \delta^1_{ji} = 0$ and constraint (1) is inactive.

Constraints (4) and (2), relative to planes landing on different runways, are linked in the same manner.

If $y_{ir} = y_{jr} = 1$, i.e. planes i and j land on the same runway r, then constraint (5) ensures that $z_{ij} = 1$. Otherwise z_{ij} will be either equal to 0 or 1. The value $z_{ij} = 0$ will activate constraint (2) which is less constraining than (1) since $S_{ij} > s_{ij}$, and zero value will be preferred in an optimization problem.

Constraint (6) ensures that each plane lands on exactly one runway.

Constraint (7) ensures that each plane lands within its time window.