European Journal of Operational Research 238 (2014) 404–414

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

The Clustered Orienteering Problem

E. Angelelli, C. Archetti *, M. Vindigni

Department of Economics and Management, University of Brescia, Italy

ARTICLE INFO

Article history:
Received 4 March 2013
Accepted 4 April 2014
Available online 13 April 2014

Keywords:
Orienteering Problem
Branch-and-cut
Tabu search

ABSTRACT

In this paper we study a generalization of the Orienteering Problem (OP) which we call the Clustered Orienteering Problem (COP). The OP, also known as the Selective Traveling Salesman Problem, is a problem where a set of potential customers is given and a profit is associated with the service of each customer. A single vehicle is available to serve the customers. The objective is to find the vehicle route that maximizes the total collected profit in such a way that the duration of the route does not exceed a given threshold. In the COP, customers are grouped in clusters. A profit is associated with each cluster and is gained only if all customers belonging to the cluster are served. We propose two solution approaches for the COP: an exact and a heuristic one. The exact approach is a branch-and-cut while the heuristic approach is a tabu search. Computational results on a set of randomly generated instances are provided to show the efficiency and effectiveness of both approaches.

1. Introduction

The class of routing problems with profits is composed by a wide variety of problems which share the same characteristic: in contrast to what happens in classical routing problems, not all customers need to be served. Instead, a profit is typically associated with each customer and the problem is to choose the right set of customers to serve satisfying a number of side constraints while optimizing a given objective function (maximize the total collected profit, minimize the traveling cost, maximize the difference among profits and costs, etc.). Among the routing problems with profits, the Traveling Salesman Problems with Profits (TSPPs) are problems where a single vehicle is available to carry out the service (see Feillet, Dejax, & Gendreau (2005) for an excellent survey on TSPPs). In Feillet et al. (2005), TSPPs are classified in three main categories, depending on the objective function and side constraints: the Orienteering Problem (OP), also known as the Selective Traveling Salesman Problem, where the objective is to find the vehicle route that maximizes the total collected profit in such a way that the route duration does not exceed a given threshold; the Prize Collecting TSP (PCTSP), which is the problem of finding the route that minimizes the traveling cost while ensuring that the profit collected is greater than or equal to a minimum requested amount; finally, the Profitable Tour Problem (PTP) which is the problem of finding the route that maximizes the difference between the total collected profit and the traveling cost. The OP is certainly the variant that has received more attention in the literature. It has been introduced in Tsiligirides (1984) and then studied in Golden, Levy, and Vohra (1987) as an application of the home fuel delivery problem. A number of heuristic algorithms have been proposed (see Chao, Golden, & Wasil, 1996; Chekuri, Korula, & Pál, 2012; Gendreau, Laporte, & Semet, 1998a; Golden et al., 1987; Golden, Wang, & Liu, 1988; Liang, Kulturel-Konak, & Smith, 2002; Tsiligirides, 1984; Wang, Sun, Golden, & Jia, 1995) and also efficient exact algorithms (see Fischetti, Salazar-González, & Toth, 1998; Gendreau, Laporte, & Semet, 1998b; Laporte & Martello, 1990; Ramesh, Yoon, & Karwan, 1992). The recent literature has been focused on variants or generalizations of the OP, especially on the multiple vehicle version, called Team Orienteering Problem (TOP). We do not survey here the literature related to this variants as it is quite wide. The reader is referred to Keller (1989), Vansteenwegen, Souffriau, and Van Oudheusden (2011) and Archetti, Speranza, and Vigo (2013) for excellent surveys on the OP and on routing problems with profits in general.

In this paper we address a generalization of the OP which we call the Clustered Orienteering Problem (COP). In this problem, customers are grouped in clusters. A profit is associated with each cluster and is collected only if all customers in the cluster are served.

The interest in studying the COP is motivated by the analysis of practical application problems that can be formulated as variants or generalizations of the COP. Examples of such applications are mainly related to the distribution of mass products, like in the case where customers are retailers belonging to different supply chains

* Corresponding author. Tel.: +39 0302988587; fax: +39 0302400925.
E-mail addresses: angele@eco.unibs.it (E. Angelelli), archetti@eco.unibs.it (C. Archetti), vindigni@eco.unibs.it (M. Vindigni).

http://dx.doi.org/10.1016/j.ejor.2014.04.006
0377-2217/

and contracts are made between the carriers and the chains. Thus, if a carrier agrees to serve a chain, he/she has to serve all retailers belonging to that chain. Another example is the case where products are divided in brands: the carrier stipulates contracts with product shippers; retailers (customers) require a certain amount of each product; in order to get the profit, the carrier has to serve all retailers requiring a certain amount of product of the brand for which he/she has a contract. Also, a different case arises when customers are clustered in areas and the profit is collected only if all customers in an area are served. This happens for example in the case of companies providing waste collection services: they can be engaged by municipalities to serve given areas and visit all customers there.

The main contribution of the paper is the introduction and the study of the COP. We give a mathematical formulation of the problem and propose two solution approaches: an exact solution approach which is a branch-and-cut algorithm, and a heuristic algorithm based on a tabu search scheme. The exact solution approach is able to solve instances with up to 318 vertices and 15 clusters or 226 vertices and 25 clusters in one hour of computing time while, on the same classes of instances, the heuristic gives high quality solutions in an extremely short amount of time. Three variants of the heuristic have been implemented and tested also on larger instances which could not be solved by the exact algorithm. The variant based on a multi-start approach proved to be the best.

The paper is organized as follows. In Section 2 we describe the problem and propose a mathematical formulation. Section 3 is devoted to the branch-and-cut algorithm, together with the valid inequalities and branching rules we propose to enhance the efficiency of the approach. In Section 4 we describe the tabu search algorithm. In Section 5 we present the computational tests we made in order to verify the effectiveness of both the exact and the heuristic algorithm and we discuss the computational results. Conclusions are drawn in Section 6.

2. Problem description and formulation

The COP can be represented by an undirected graph $G = (V, E)$ where V is the set of vertices and E is the set of edges. Set $V = \{v_0, v_1, \dots, v_n\}$ is formed by vertex v_0 which is the depot where the vehicle starts and ends its tour and vertices $v_1, \dots, v_n$ which are the customers. A cover $S = \{S_1, S_2, \dots, S_k\}$ of $V \setminus \{0\}$ is defined where $V \setminus \{0\} = \cup_{i=1}^{k} S_i$. In the following we call each element $S_i \in S$ a *cluster*. Each customer belongs to at least one cluster $S_i, i = 1, \dots, k$. Note that a customer can belong to more than one cluster. An integer value p_i is associated with each cluster S_i and corresponds to the profit which is collected only if all customers in S_i are served (visited) by the vehicle. A cost t_e is related to each edge $e \in E$ and represents the time needed to traverse edge e. We assume that travel times satisfy the triangle inequality. A single vehicle is available and a maximum time limit T_{max} is imposed on the duration of the vehicle route. Note that, it is not necessary that all vertices belonging to a cluster are visited consecutively, i.e., the vehicle can start visiting some vertices in a cluster, leave the cluster and visit vertices belonging to different clusters and then visit the remaining vertices of the previous cluster. The objective is to find the route that maximizes the total collected profit and such that the duration is lower than or equal to T_{max}. Note that, if all clusters are formed by single customers, the COP reduces to the OP.

In order to give a mathematical formulation of the problem let us first introduce the following notation:

- $\delta(U)$: set of edges with one endpoint in U and one endpoint in $V \setminus U$. For the ease of notation, we will write $\delta(j)$ for the set of edges adjacent to the single vertex v_j.
- $E(U)$: set of edges with both endpoints in $U \subseteq V$.
- z_i: binary variable equal to 1 if all customers in cluster $S_i \in S$ are served, 0 otherwise.
- y_j: binary variable equal to 1 if vertex $v_j \in V$ is served, 0 otherwise.
- x_e: binary variable equal to 1 if edge $e \in E$ is traversed, 0 otherwise.

The COP can then be formulated as follows:

$$\max \sum_{S_i \in S} p_i z_i \tag{1}$$

$$y_0 = 1 \tag{2}$$

$$\sum_{e \in \delta(j)} x_e = 2y_j \qquad \forall v_j \in V \tag{3}$$

$$\sum_{e \in E} t_e x_e \leqslant T_{\max} \tag{4}$$

$$\sum_{e \in E(U)} x_e \leqslant \sum_{v_j \in U \setminus \{v_t\}} y_j, \qquad \forall U \subseteq V \setminus \{0\}, \quad \forall v_t \in U \tag{5}$$

$$z_i \leqslant y_j, \qquad \forall S_i \in S, \quad \forall v_j \in S_i \tag{6}$$

$$z_i \in \{0, 1\} \qquad \forall S_i \in S \tag{7}$$

$$x_e \in \{0, 1\} \qquad \forall e \in E \tag{8}$$

$$y_j \in \{0, 1\} \qquad \forall v_j \in V. \tag{9}$$

The objective function (1) aims at maximizing the total collected profit. Constraint (2) imposes to visit the depot while (3) establishes to traverse two edges adjacent to each served vertex. Inequality (4) imposes the maximum time limit on the route duration while (5) are the subtour elimination constraints. (6) imposes that all vertices belonging to a cluster must be served in order to get the corresponding profit. Finally, (7)–(9) are variable definitions.

Note that this formulation is an adaptation of the formulation proposed in Fischetti et al. (1998) for the solution of the OP. The branch-and-cut algorithm proposed in Fischetti et al. (1998) is, to the best of our knowledge, the best exact solution approach proposed in the literature for the solution of the OP. Thus, we are confident that the formulation we propose will be also effective for the COP. The differences with respect to the standard OP formulation are related to the presence of the z variables indicating whether a cluster S_i is served. As a consequence, while in the OP the sum of the profits of the served customers is maximized, in COP we maximize the sum of the profits of the clusters served. Moreover, constraints (6) and (7) are not present in the OP.

In the following section we present a branch-and-cut algorithm for the exact solution of the COP.

3. A branch-and-cut algorithm

We implemented a branch-and-cut algorithm in order to solve model (1)–(9) which we call *COP-CUT*. At each node of the branch-and-bound tree we solve the linear relaxation of (1)–(9) where subtour elimination constraints (5) are originally removed from the formulation and inserted only once violated. In the following we describe the valid inequalities and the branching rules implemented in order to improve the efficiency of the algorithm, together with the separations algorithms which detect violated valid inequalities and subtour elimination constraints.

3.1. Valid inequalities

In order to strengthen the formulation, we introduced different valid inequalities. The first class of valid inequalities, which we call *logical constraints*, is the following: