European Journal of Operational Research 238 (2014) 451–457

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

The static bicycle relocation problem with demand intervals

Güneş Erdoğan [a,*], Gilbert Laporte [b], Roberto Wolfler Calvo [c]

[a] School of Management, University of Southampton, Highfield, Southampton SO17 1BJ, United Kingdom
[b] Canada Research Chair in Distribution Management, HEC Montréal, 3000 chemin de la Côte-Sainte-Catherine, Montreal H3T 2A7, Canada
[c] LIPN, Université Paris 13, Paris, France

ARTICLE INFO

Article history:
Received 21 January 2013
Accepted 8 April 2014
Available online 18 April 2014

Keywords:
Traveling salesman
Pickup and delivery
Branch-and-cut

ABSTRACT

This study introduces the *Static Bicycle Relocation Problem with Demand Intervals* (SBRP-DI), a variant of the *One Commodity Pickup and Delivery Traveling Salesman Problem* (1-PDTSP). In the SBRP-DI, the stations are required to have an inventory of bicycles lying between given lower and upper bounds and initially have an inventory which does not necessarily lie between these bounds. The problem consists of redistributing the bicycles among the stations, using a single capacitated vehicle, so that the bounding constraints are satisfied and the repositioning cost is minimized. The real-world application of this problem arises in rebalancing operations for shared bicycle systems. The repositioning subproblem associated with a fixed route is shown to be a minimum cost network problem, even in the presence of handling costs. An integer programming formulation for the SBRP-DI are presented, together with valid inequalities adapted from constraints derived in the context of other routing problems and a Benders decomposition scheme. Computational results for instances adapted from the 1-PDTSP are provided for two branch-and-cut algorithms, the first one for the full formulation, and the second one with the Benders decomposition.

1. Introduction

The *Static Bicycle Relocation Problem with Demand Intervals* (SBRP-DI) is defined on a complete directed graph $G = (V, A)$, where $V = \{0, \ldots, n\}$ is the set of vertices and A is the set of arcs. Vertex 0 is called the *depot* while the remaining vertices are called *stations*. Associated with each vertex $i \in V$, are three parameters (l_i, b_i, u_i) corresponding respectively to the lower bound, the current supply, and the upper bound of the feasible amount of a commodity at the vertex. With each arc $(i,j) \in A$ is associated a travel cost t_{ij}, and the cost associated with the handling of a bicycle is denoted as h. A vehicle of capacity Q leaves the depot, performs a tour visiting each vertex at most once to perform a pickup or a delivery, and returns to the depot. At the end of the tour, the resulting inventory at every vertex i must lie within the interval $[l_i, u_i]$. The objective is to minimize the total travel and handling cost.

The SBRP-DI is a variant of the *One-Commodity Pickup and Delivery Traveling Salesman Problem* (1-PDTSP) studied by Hernández-Pérez and Salazar-González (2004, 2007), in which a single capacitated vehicle visits customers to pick up and deliver the same commodity. In addition to the demand intervals, there are a number of features of the SBRP-DI that differentiate it from the 1-PDTSP. Firstly, the tour may not visit all vertices. Secondly, the commodity cannot flow through the depot. Lastly, a handling cost h is added to the routing cost per every commodity unit handled. The demand intervals introduce a degree of flexibility associated with *transshipment vertices*, i.e. vertices $i \in V$ for which $l_i \leqslant b_i \leqslant u_i$. A transshipment vertex may or may not be visited, which may help decrease the cost of routing by supplying or demanding commodities as required. Notably, the *Swapping Problem* (SP) introduced by Anily and Hassin (1992) also involves pickups and deliveries with transshipment vertices and multiple commodities. Branch-and-cut algorithms for the SP were developed by Bordenave, Gendreau, and Laporte (2009, 2012) and Erdoğan, Cordeau, and Laporte (2010). Bordenave, Gendreau, and Laporte (2010) have also developed construction and improvement heuristics for the SP.

Real-world applications of the SBRP-DI arise in shared bicycle systems, which have attracted the attention of several groups of researchers in recent years. These have been studied from several perspectives: evaluating the mobility patterns of users, determining the number and location of stations, maximizing user satisfaction, and minimizing the cost of relocating the bicycles. The study by Raviv, Tzur, and Forma (2013) categorizes the bicycle relocation problems as *static* and *dynamic*, which occur when the system activity level is low and high, respectively. The authors study the

* Corresponding author. Tel.: +44 23 8059 8882.
E-mail addresses: G.Erdogan@soton.ac.uk (G. Erdoğan), gilbert.laporte@cirrelt.ca (G. Laporte), Roberto.Wolfler@lipn.univ-paris13.fr (R. Wolfler Calvo).

http://dx.doi.org/10.1016/j.ejor.2014.04.013
0377-2217/

Static Repositioning Problem (SRP), in which the objective is to minimize a convex nonlinear function representing user dissatisfaction, and present four integer programming formulations for the SRP, together with computational results.

Research on shared vehicle systems is becoming increasingly widespread. *Static Stations Balancing Problem* is studied in Benchimol et al. (2011), and a 9.5-approximation algorithm is provided for this problem. Another closely related problem is that of Chemla, Meunier, and Wolfler Calvo (2013), where a bicycle station can be visited more than once for a pickup or a delivery. The authors provide a branch-and-cut algorithm as well as a tabu search algorithm for this problem. A variable neighborhood search algorithm for balancing bicycle sharing systems is designed in Rainer-Harbach, Papazek, Hu, and Raidl (2013). An interesting study by Schuijbroek, Hampshire, and van Hoeve (2013) employs queuing analysis to determine service level requirements at each bike sharing station, and to solve the resulting vehicle routing problem. A recent study (Nair & Miller-Hooks, 2014) analyzes the equilibrium network design problem of shared-vehicle systems and presents a bi-level, mixed-integer program.

Most of the problems cited above are based on a single demand or supply value for every customer, which restricts the vehicle to picking or delivering a preset number of commodities. The SBRP-DI is therefore more general, and empirically more difficult, because these values must lie within an interval. It is a special case of the models presented in Raviv et al. (2013) since these authors use a convex user dissatisfaction function which can be set to zero inside the interval and to infinity outside it. Let us define the *deficit* of station i as $d_i = \max\{l_i - b_i, 0\}$ and its *excess* as $e_i = \max\{b_i - u_i, 0\}$. Transforming an instance of the SBRP-DI into one of the 1-PDTSP by setting the demand (supply) of a station to be equal to its deficit (excess) may yield an infeasible instance since the sum of the demands may not be equal to the sum of supplies.

In this study, we provide two exact algorithms for the SBRP-DI. To gain a better insight into the SBRP-DI, we first study the subproblem of computing pickup and delivery quantities when the vehicle route is fixed. We show that this subproblem is a minimum cost network flow problem (MCNFP), whether the handling cost $h = 0$ or not. We also present a model for the general problem consisting of simultaneously determining the vehicle route as well as the pickup and delivery quantities. We develop a standard branch-and-cut algorithm as well as a Benders decomposition based branch-and-cut algorithm, and we present computational results for both cases.

The remainder of this paper is organized as follows. In Section 2, we study the subproblem corresponding to a fixed route, without and with handling cost. In Section 3 we present an integer linear programming formulation for the SBRP-DI based on our findings in Section 2, as well as valid inequalities we have adapted from the routing literature. A Benders decomposition scheme for the integer linear programming formulation is provided in Section 4, together with a unified branch-and-cut algorithm capable of handling both algorithms. This is followed by computational results in Section 5, and by conclusions in Section 6.

2. The fixed route subproblem

We first focus on the subproblem of determining the pickup and delivery amounts when the vehicle route is fixed. For the sake of simplicity, we assume that the vertices are numbered in the order they are visited. The two cases for which $h = 0$ and $h \geqslant 0$ will be treated separately.

2.1. The fixed route subproblem with no commodity handling cost

The fixed route subproblem without handling cost is called the SBRP-DIF and is defined on an auxiliary graph $\widehat{G} = (\widehat{V}, \widehat{A})$. Denote the set of vertices by $\widehat{V} = \widehat{V}_1 \cup \widehat{V}_2$, with $\widehat{V}_1 = V$ and $\widehat{V}_2 = \{n+1\}$. The supply of vertex $i \in \widehat{V}_1$ is $\hat{b}_i = b_i$, and $\hat{b}_{n+1} = -\sum_{i \in V} b_i$. Denote the set of arcs by $\widehat{A} = \widehat{A}_1 \cup \widehat{A}_2$, where $\widehat{A}_1$ and $\widehat{A}_2$ are constructed as follows. For every vertex $i \in \widehat{V}_1 \setminus \{0, n\}$, insert an arc $(i, i+1)$ into $\widehat{A}_1$, with cost 0, lower bound 0 and upper bound Q. These arcs represent the number of units transported to the next vertex. We also insert two arcs $(0,1)$ and $(n,0)$ into $\widehat{A}_1$, with cost 0, lower bound 0 and upper bounds $\min\{b_0, Q\}$ and Q, respectively. These arcs represent number of units leaving and entering the depot. The flow on the arc $(0,1)$ is also bounded above by the supply at the depot, in order to avoid commodities from flowing through the depot. For every vertex $i \in \widehat{V}_1$, insert an arc $(i, n+1)$ into $\widehat{A}_2$, with cost 0, lower bound l_i and upper bound u_i. This arc represents the final amount of the commodity left at vertex i. Define the set of arcs leaving vertex i as $\delta^+(i)$, and the set of arcs entering vertex i as $\delta^-(i)$. Let z_{ij} denote the commodity flow on arc (i,j). We write $z(S)$ to denote the sum of the z variables in arc set S, i.e. $z(S) = \sum_{(i,j) \in S} z_{ij}$. We then have to solve

$$\text{(SBRP-DIF)} \quad z(\delta^+(i)) - z(\delta^-(i)) = \hat{b}_i \quad (i \in \widehat{V}) \tag{1}$$

$$l_i \leqslant z_{ij} \leqslant u_i \quad ((i,j) \in \widehat{A}_2) \tag{2}$$

$$0 \leqslant z_{01} \leqslant \min\{b_0, Q\} \tag{3}$$

$$0 \leqslant z_{ij} \leqslant Q \quad ((i,j) \in \widehat{A}_1 \setminus \{(0,1)\}). \tag{4}$$

Fig. 1 depicts an instance of the SBRP-DIF. Using an enhanced capacity scaling algorithm, together with the fact that the number of arcs is $O(n)$, the problem stated above can be solved in $O(n^2 \log n^2)$ time (Ahuja, Magnanti, & Orlin, 1993).

2.2. The fixed route subproblem with commodity handling cost

The fixed route subproblem with handling cost is called the SBRP-DIHF and is defined on an auxiliary graph $\overline{G} = (\overline{V}, \overline{A})$. Denote the set of vertices by $\overline{V} = \overline{V}_1 \cup \overline{V}_2 \cup \overline{V}_3$, with $\overline{V}_1 = V$ and $\overline{V}_3 = \{2n+2\}$. We construct $\overline{V}_2$ by including a vertex for every vertex $i \in V$, where the copy of vertex i in $\overline{V}_2$ is $n+1+i$. Set the supply of vertex $i \in \overline{V}_1$ as $\bar{b}_i = b_i$ and the demand of vertex $n+1+i \in \overline{V}_2$ as $\bar{b}_{n+1+i} = -b_i$. Let $\bar{b}_{2n+2} = 0$. Denote the set of arcs $\overline{A} = \overline{A}_1 \cup \overline{A}_2 \cup \overline{A}_3$, where $\overline{A}_1$, $\overline{A}_2$, and $\overline{A}_3$ are constructed as follows. For vertices $i \in \overline{V}_1 \setminus \{0, n\}$, insert an arc $(i, i+1)$ into $\overline{A}_1$, with cost 0, lower bound 0 and upper bound Q. These arcs represent the number of units transported to the next vertex. We also insert two arcs $(0,1)$ and $(n,0)$ into $\overline{A}_1$, with cost h, lower bound 0 and

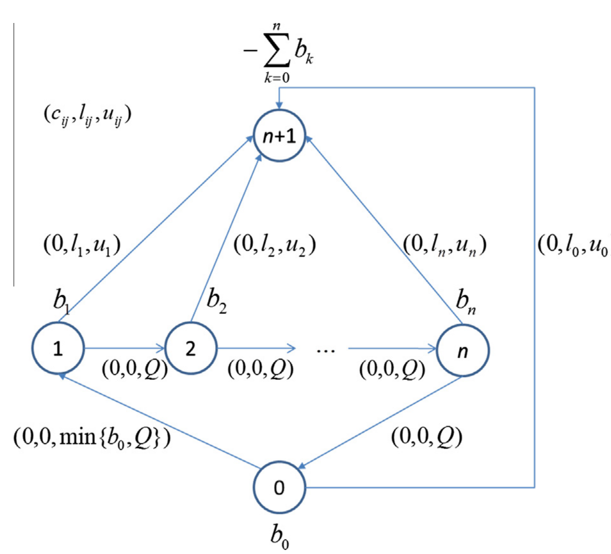

Fig. 1. Instance of the SBRP-DIF.