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Decision Support

A synergy of multicriteria techniques to assess additive value models

Christian Hurson [a], Yannis Siskos [b,*]

[a] IAE, Université de Rouen, 3, Avenue Pasteur, F-76186 Rouen Cedex, France
[b] University of Piraeus, 80, Karaoli & Dimitriou Street, GR-18534 Piraeus, Greece

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ABSTRACT

The assessment of additive value functions in Multicriteria Decision Aid (MCDA) has to face issues of legitimacy and technical difficulties when real decision makers are involved. This paper presents a synergy of three complementary techniques to assess additive models on the whole criteria space. The synergy includes a revised MACBETH technique, the standard MAUT trade-off analysis and UTA-based methods for the assessment of both the marginal value functions and the weighting factors. The paper uses a set of original robustness measures and rules associated with revised MACBETH and UTA in order to manage multiple linear programming solutions and to extract robust conclusions from them. Finally, to illustrate the methods' synergy, an application example is presented, dealing with the planning of metro extension lines.

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1. Introduction and background

Today, the additive value model is the most popular one for Multicriteria Decision Aid (MCDA) activities, especially when a decision analyst wishes to obtain a complete ranking of a set of actions evaluated on a consistent family of criteria. The assessment of additive value functions has to face issues of legitimacy and technical difficulties when real decision makers are involved.

There is a plethora of classical well known techniques to assess additive value functions on the whole criteria space based on Multi-attribute Utility Theory (MAUT). For a detailed presentation of these techniques, see for instance Fishburn (1967), Keeney and Raiffa (1976), Keeney (1980, 1992), and Farquhar (1984). Nevertheless, this kind of technique, which requires explicit trade-offs between economic, political and social criteria, seems to be difficult to implement in real world decision environments, usually because of the amount of cognitive effort needed, or for ethical reasons.

A promising technique to overcome some of the above difficulties is the MACBETH method (Measuring Attractiveness by a Categorical Based Evaluation Technique) proposed by Bana e Costa and Vansnick (1994, 1997). Using MACBETH, a single Decision Maker (DM) is aided in an interactive way to obtain a global and additive evaluation of a set of actions A from their evaluations on multiple criteria. Firstly, the method estimates the marginal values of the actions for every separate criterion (on a scale from 0 to 100). These values are interpreted in terms of the actions' attractiveness. In fact, the DM has to make a pairwise comparison of all the actions, for each criterion separately, on a pure ordinal scale of attractiveness. Secondly, using a similar mode of questioning to compare the criteria, their relative importance (weights) are determined. MACBETH has been extensively applied in different domains of management (see Bana e Costa & Vansnick, 1997). An up-to-date comprehensive overview of the MACBETH approach to MCDA is recently published by Bana e Costa, De Corte, and Vansnick (2012).

The MACBETH computational procedure corrects many undesirable difficulties of Saaty's well-known AHP method thanks to linear programming techniques (cf. Bana e Costa & Vansnick, 2008). However, because the method requires pairwise comparisons between actions, it is unable to handle large sets of actions such as when assessing stocks or combinations of stocks in a stock market. Another difficulty is that it is not possible to assess the decision model on the whole criteria space. In order to overcome these difficulties a new version of MACBETH is proposed in this paper.

Furthermore, since the weights involved in an additive value model are trade-offs between criteria attractiveness that are preference-independent and constant, the determination of the criteria weights faces issues of legitimacy (see Keeney & Raiffa, 1976, for instance). The estimated weights (trade-offs) may not in fact be preference-independent and constant – or indeed the DM may refuse a reasoning based on trade-offs between certain criteria. These difficulties are addressed by the other methods in our synergy.

* Corresponding author. Tel.: +30 2104142260.
E-mail addresses: christian.hurson@univ-rouen.fr (C. Hurson), ysiskos@unipi.gr (Y. Siskos).

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On the other hand, important research efforts have gone into the inference of additive value functions from global preference structures. This paradigm is the disaggregation or ordinal regression approach initiated by the well-known UTA methods of Jacquet-Lagrèze and Siskos (1982, 2001). For an overview of UTA methods, see Siskos, Grigoroudis, and Matsatsinis (2005). The preference disaggregation approach refers to the analysis (disaggregation) of the global preferences (judgment policy) of the DM in order to identify the criteria aggregation model that underlies the preference result. Contrary to the traditional aggregation paradigm, where the criteria aggregation model is known a priori and the global preference is unknown, the philosophy of preference disaggregation aims to infer the preference models from given global preferences. The goal of this approach is to support the DM to improve her(his) knowledge about the decision making problem and her(his) way of preferring in order to allow a consistent decision to be achieved.

UTA-based methods include robustness analysis to take account of the gap between the DM's "true" model and the model resulting from the disaggregation computational mechanism. Roy (2010) considers robustness as an enabling tool for decision analysts to resist the phenomena of approximations and ignorance zones. It should be emphasised that robustness refers mainly to the decision model, in the light of the assertion "robust models produce a fortiori robust results". However, robustness should also refer to the results and the decision support activities (e.g. conclusions, argumentation). In UTA methods robustness uses LP as the main inference mechanism. In this spirit several UTA-type methods have been developed such as UTA^{GMS} (Greco, Mousseau, & Słowiński, 2008), GRIP (Figueira, Greco, & Słowiński, 2009), and RUTA (Kadzinski, Greco, & Slowiński, 2013) to provide the DM with robust conclusions, Extreme Ranking Analysis (Kadzinski, Greco, & Slowiński, 2012a) to determine the extreme ranking positions taken by the actions, and finally the robustness measurement control based on Monte Carlo sampling techniques (stochastic ordinal regression, see Kadzinski & Tervonen, 2013a, 2013b; entropy measurement control, see Greco, Siskos, & Słowiński, 2012).

This paper presents a synergy of three complementary techniques to assess additive models on the whole criteria space which includes a revised MACBETH technique, the standard MAUT trade-off analysis and UTA-based methods for the assessment of both the marginal value functions, which are piecewise linear, and the weighting factors. The paper also uses a set of robustness measures and rules associated with MACBETH and UTA, in order to manage multiple linear programming solutions and extract robust conclusions from them.

The rest of the paper is organised as follows: The additive value model and its legitimacy conditions are presented in Section 2 while the MACBETH-MAUT-UTA synergy is outlined in Section 3. Section 4 presents a set of cardinal measures of robustness and visualisation rules associated with MACBETH and UTA. Section 5 illustrates the methodology via a case application dealing with planning of metro extension lines. Section 6 concludes the paper. Finally, a brief presentation of MACBETH technique and a part of numerical results are respectively presented in Appendices A and C, while UTA II method is sketched in Appendix B.

2. An anatomy of the additive value model

A multicriteria value function is supposed to be additive if it has the following form:

$$u(\mathbf{g}) = \sum_{i=1}^{n} p_i u_i(g_i) \tag{1}$$

under the following normalisation constraints:

$$\sum_{i=1}^{n} p_i = 1 \tag{2}$$

$$u_i(g_{i^*}) = 0, u_i(g_i^*) = 1 \quad \forall i = 1, 2, \ldots, n \tag{3}$$

where u_i, $i = 1,2,\ldots,n$ represent the marginal non decreasing value functions defined on the respective criteria g_i; g_{i^*} and g_i^* are respectively the worst and the best evaluation level of the criterion g_i; $\mathbf{g} = (g_1,g_2,\ldots,g_n)$ is the multicriteria evaluation vector; and p_i is the relative (positive) weight of the function u_i. In MACBETH method (presented in Appendix A) the functions u_i are normalised between 0 and 100, i.e. $u_i(g_i^*) = 100$ for every i.

For every pair of actions a and b from a set of actions A, with respective multicriteria evaluations on the n criteria $\mathbf{g}(a)$ and $\mathbf{g}(b)$, the value function u must verify the following properties:

$$\forall (a,b) \in A : \begin{cases} u[g(a)] > u[g(b)] \iff a \succ b & (preference) \\ u[g(a)] = u[g(b)] \iff a \sim b & (indifference) \end{cases} \tag{4}$$

The necessary hypothesis to validate an additive value function is the preference independence of the criteria (see for instance Bouyssou & Pirlot, 2005; Keeney & Raiffa, 1976; Keeney, 1980, 1992). Concerning the weighting factors p_i, these inter-criteria parameters must be constant substitution rates or trade-offs between u_i and they must be assessed accordingly.

According to the common definition a substitution rate or trade-off s_{ir}^{g} between the criterion g_i and a reference criterion g_r is the amount of units that must be gained on criterion g_r to the evaluation vector $\mathbf{g}$ in order to compensate exactly the loss of one unit of the criterion g_i. Consequently s_{ir}^{g} is defined in such a way that the following fictitious actions are indifferent:

$$(g_1, g_2, \ldots, g_i, \ldots, g_r, \ldots, g_n) \sim (g_1, g_2, \ldots g_i - 1, \ldots, g_r + s_{ir}^{\mathbf{g}}, \ldots, g_n) \tag{5}$$

When $u(\mathbf{g})$ is differentiable this definition could be written as follows:

$$s_{ir}^{\mathbf{g}} = \frac{\dfrac{\partial u(\mathbf{g})}{\partial g_i}}{\dfrac{\partial u(\mathbf{g})}{\partial g_r}} \tag{6}$$

In fact the relations (5) and (6) are equivalent. It holds:

$$du(\mathbf{g}) = \sum_{i=1}^{n} \frac{\partial u(\mathbf{g})}{\partial g_i} dg_i \tag{7}$$

from which, by applying (5):

$$\frac{\partial u(\mathbf{g})}{\partial g_r} s_{ir}^{\mathbf{g}} - \frac{\partial u(\mathbf{g})}{\partial g_i} = 0 \tag{8}$$

which gives the relation (6).

The trade-off vector to the multicriteria evaluation vector $\mathbf{g}$ is the following line vector (9) which is collinear of the gradient of u:

$$s_r^{\mathbf{g}} = (s_{1r}^{\mathbf{g}}, s_{2r}^{\mathbf{g}}, \ldots, s_{ir}^{\mathbf{g}}, \ldots, 1, \ldots, s_{nr}^{\mathbf{g}}) \tag{9}$$

Of course the rth component is equal to 1 ($s_{rr}^{\mathbf{g}} = 1$).

The decision model for a single DM is supposed to be an additive value function if and only if the trade-offs s_{ir}^{g} between g_i and g_r, for every i, are independent of the values taken by the criteria within the vector $\mathbf{g}$ (condition of preference independence). On the other hand, these trade-offs must remain constant if they are considered as trade-offs between the marginal values $u_i(g_i)$ within the weighted sum model (1).

3. A synergy of complementary methods

The methodology proposed in this paper is based on a synergy of three complementary approaches: MACBETH, MAUT and UTA, as

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