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Invited Review

Electricity swing option pricing by stochastic bilevel optimization: A survey and new approaches

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ABSTRACT

We demonstrate how the problem of determining the ask price for electricity swing options can be considered as a stochastic bilevel program with asymmetric information. Unlike as for financial options, there is no way for basing the pricing method on no-arbitrage arguments. Two main situations are analyzed: if the seller has strong market power he/she might be able to maximize his/her utility, while in fully competitive situations he/she will just look for a price which makes profit and has acceptable risk. In both cases the seller has to consider the decision problem of a potential buyer – the valuation problem of determining a fair value for a specific option contract – and anticipate the buyer's optimal reaction to any proposed strike price. We also discuss some methods for finding numerical solutions of stochastic bilevel problems with a special emphasis on using duality gap penalizations.

1. Introduction

Swing options are derivative contracts – usually between a producer or wholesaler, and a retailer – which give their purchasers the right to buy the underlying commodity at a prespecified exercise price per unit during a future term of validity. For each decision period within this timeframe the purchaser is allowed to choose the quantities delivered within prespecified bounds. Swing contracts are widely used in energy related commodity markets. In this paper we are particularly interested in electricity swing options.

Two important quantification questions arise: *pricing* aims at finding an exercise price such that neither the holder nor the seller of the option are disadvantaged. On the other hand, *valuation* of an existing contract with given exercise price should result in a fair resale price.

The valuation problem can be described (see e.g. Haarbrücker and Kuhn, 2009) by an optimization problem for the buyer, where the optimal value corresponds to the economic value of the swing option. The seller however has a different objective than the buyer: he/she tries to find an optimal offered price with respect to his/her own specification, but also needs to consider the buyer's reaction to this offer (e.g. Broussev and Pflug, 2009). Since both agents take optimal decisions which are related to each other, the very nature of the pricing problem is bilevel and of the leader–follower type: the option seller plays the role of the leader, or upper level decision maker, when setting the price, but he/she acts in view of the reaction of the option buyer, who is the follower, or lower level decision maker. Throughout this paper, we mark all upper level decision problems of the seller by [*UL*] and those of the lower level option buyer by [*LL*]. Since future spot market prices are not known at the time of contracting, and since the buyer may make decisions at several time steps, we have to deal with a multistage stochastic bilevel optimization problem.

For this reason we will review swing option modeling as well as stochastic bilevel programming and introduce some new solution methods in the following. While bilevel programming and in particular stochastic (two stage) bilevel programming has recently made considerable advances, stochastic decision problems in electricity usually are large (in particular if formulated as stochastic optimization problems defined on tree structures) and have certain nonstandard features. This means that standard algorithms cannot be applied or lead to unacceptable computation times.

This is true even for the simplest setup, where the optimization problems of both agents are linear, given the decisions of the other level, but bilinear if all decision variables are considered at once. On the other hand the swing option problem is simpler than other typical bilevel problems in that all decisions are determined by the seller's decision for a strike price, which is a single number. The article will predominantly discuss such algorithms that are able to deal with the complicated overall structure of the problem, but are also suitable to exploit this special feature.

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The paper is organized as follows: Section 2 reviews some basic facts of electricity markets, describes the swing option problem from both, the buyer's and the seller's view and states it as a multistage stochastic bilevel optimization problem. Section 3 gives an overview of general solution methods for bilevel problems and introduces two simple approaches, well suited for the bilevel formulation described before. The section closes with a numerical example on swing option pricing. In the appendices we add some technical details on acceptability functionals.

2. Electricity prices and swing options

Because of the non-storability of electricity there is a layered structure of short term markets, reaching from day ahead to real time. In addition, also forward contracts exist in most electricity markets, with a gradual transition from forwards with maturities up to 2 years to the day ahead spot market. Forward contracts fix a price for future power delivery (the strike price) over a specified period in the future.

Usually, such contracts are traded over-the-counter (OTC) as forward contracts, or in standardized form. In the latter case they are called *futures*. While settlement at electricity spot and OTC markets usually is physical, futures markets mostly rely on financial settlement. We will not distinguish between forward contracts and futures throughout the rest of the paper. In fact, there is no difference in the valuation of the two contract types, if the interest rate is considered as deterministic.

Given the non-storability of electric energy, the usual no-arbitrage arguments for pricing forward contracts in financial and commodity markets cannot be applied for power markets. A functional relation between the actual spot price S_t and forward prices is not observable. However futures prices $F_{0,t_{begin},t_{end}}$ (i.e. the strike price of a futures contract with delivery between point in time t_{begin} and t_{end}, agreed at time 0) are related to expected prospective spot prices $\mathbb{E}(S_t)$ with $t_{begin} \leqslant t \leqslant t_{end}$. Stoft (2002) proposes to use the simple relation

$$F_{0,t_{begin},t_{end}} = \sum_{t=t_{begin}}^{t_{end}} \mathbb{E}[S_t], \tag{2.1}$$

where electricity is delivered between times t_{begin} and t_{end}. Others (e.g. Bessembinder and Lemmon, 2002; Geman and Vasicek, 2001) have made the effort to extend relation (2.1), which leads to

$$F_{0,t_{begin},t_{end}} = \sum_{t=t_{begin}}^{t_{end}} \mathbb{E}[S_t] + \text{risk premium } (0, t_{begin}, t_{end}). \tag{2.2}$$

Based on real data, Geman (2006) shows that the risk premium is positive if t_{begin} is small, particularly if it corresponds to a winter or summer month, and may be negative if t_{begin} is large, i.e. several years. Similar results are discussed in Giacometti et al. (2010).

Other models for the spot price and/or the forward price structure – in fact the whole arsenals of econometrics and finance – have been used as well. E.g., starting with Pilipović (1998), mean reverting Pilipović spot price models with different number of risk factors have been formulated. See Eydeland and Wolyniec (2003) for a broad overview of spot and future price models.

2.1. Swing options

Swing options – an important form of flexible delivery contracts – can be considered as the simplest and most important types of option-like electricity contracts. They are also known as *flexible nomination contracts*, *take-or-pay contracts*, or *virtual power plants*. See e.g. Kaminski and Gibner (1995), Barbieri and Garman (1996), Pilipović and Wengler (1998), and Pilipović (2007).

A swing option gives its buyer (or option holder) the right to get a commodity at a price K per unit, which is fixed now, but allows to choose the actual purchase quantities in a later moment of time. For electricity swing options we will state the price as K EUR per megawatt hour. For simplicity we will assume that delivery takes place at equidistant periods $t \in \{1, 2, \ldots, T\}$. The actual demand has to be specified one period (usually a day) before delivery. For the tth period, the demanded and delivered amount of energy (megawatt hour) is denoted by y_t, where $t \in \mathcal{T} = \{0, 1, \ldots, T-1\}$.

Usually, demands for each period (often expressed by maximum deviations from a base-line schedule) and the quantity bought over the full contract period must lie within certain bounds. Such volume constraints can be expressed as

$$\underline{e}_t \leqslant y_t \leqslant \overline{e}_t \quad \forall t \in \mathcal{T} \tag{2.3}$$

$$\underline{E} \leqslant \sum_{t=0}^{T-1} y_t \leqslant \overline{E} \tag{2.4}$$

Here $\underline{e}_t, \overline{e}_t$ denote the lower and the upper bound of energy consumption y_t for the tth period, while $\underline{E}, \overline{E}$ denote the overall lower and upper bounds for the whole contract duration. Sometimes those hard constraints are replaced by penalty payments for exceeding $\overline{e}_t, \overline{E}$ or falling below $\underline{e}_t, \underline{E}$.

If we map the index 0 to a point in time t_{begin} and introduce a time increment Δt and a point in time $t_{end} = t_{begin} + T \cdot \Delta t$, we can also handle swing options related to the contract period $(t_{begin}, t_{end}]$ and with different decision intervals Δt, for instance hourly decisions. In this case the notification time before delivery has to be $h \cdot \Delta t$ with $h = 24$.

Note that often (2.3) is expressed in terms of power (MW) fixed at the beginning of a period, so in principle we could base all decisions on power v_t and substitute

$$y_t = \Delta t \cdot v_t. \tag{2.5}$$

A possible additional condition are ramping constraints with *ratchets* ϱ_t

$$|v_t - v_{t-1}| \leqslant \varrho_t \qquad t = 1, \ldots, T-1, \tag{2.6}$$

limiting the changes in power demanded between consecutive periods. In this case p_0 has to be agreed as an additional parameter of the contract. With given time increments Δt, such constraints can easily be reformulated as linear constraints on energy:

$$-\varrho_t \cdot \Delta t \leqslant y_t - y_{t-1} \leqslant \varrho_t \cdot \Delta t. \tag{2.7}$$

Constraints of this type can be important, if the period lengths are short (e.g. hours).

We see that – unlike typical financial options – swing options are flexible with respect to time and quantity and so is able to reduce both, volume risk and price risk for the buyer. In both, the gas and the electricity industry swing options have been used for many years, either as embedded options related to general delivery contracts, or in the form of separate contracts. Typical buyers in the electricity sector are public distributors with fixed retail prices, facing random load and spot prices. See e.g. Doege et al. (2006) regarding the usage of swing options as hedging instruments.

Swing contracts usually are traded bilaterally and hence may be subject to market power: often the seller of a contract will be a big producer or even a big, state owned entity. On the other hand, if market efficiency and liquidity are high, the seller's ability to set the price might be severely limited. We will consider both cases in the following. In any case, the seller's scope is limited by the buyer's possibility to buy the commodity at the spot price if the swing-price is too high.