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An evaluation of semidefinite programming based approaches for discrete lot-sizing problems ☆

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ABSTRACT

The present work is intended as a first step towards applying semidefinite programming models and tools to discrete lot-sizing problems including sequence-dependent changeover costs and times. Such problems can be formulated as quadratically constrained quadratic binary programs. We investigate several semidefinite relaxations by combining known reformulation techniques recently proposed for generic quadratic binary problems with problem-specific strengthening procedures developed for lot-sizing problems. Our computational results show that the semidefinite relaxations consistently provide lower bounds of significantly improved quality as compared with those provided by the best previously published linear relaxations. In particular, the gap between the semidefinite relaxation and the optimal integer solution value can be closed for a significant proportion of the small-size instances, thus avoiding to resort to a tree search procedure. The reported computation times are significant. However improvements in SDP technology can still be expected in the future, making SDP based approaches to discrete lot-sizing more competitive.

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1. Introduction

Capacitated lot-sizing arises in industrial production planning whenever changeover operations such as preheating, tool changing or cleaning are required between production runs of different products on a machine. The amount of the related changeover costs usually does not depend on the number of products processed after the changeover. Thus, to minimize changeover costs, production should be run using large lot sizes. However, this generates inventory holding costs as the production cannot be synchronized with the actual demand pattern: products must be held in inventory between the time they are produced and the time they are used to satisfy customer demand. The objective of lot-sizing is thus to reach the best possible trade-off between changeover and inventory holding costs while taking into account both the customer demand satisfaction and the technical limitations of the production system.

An early attempt at modelling this trade-off can be found in Wagner and Whitin (1958): the authors consider the problem of planning production for a single product on a single resource with an unlimited production capacity. Since this seminal work, a large part of the research on lot-sizing problems has focused on modelling operational aspects in more detail to answer the growing industry need to solve more realistic and complex production planning problems. An overview of recent developments in the field of modelling industrial extensions of lot-sizing problems is provided in Jans and Degraeve (2008).

In the present paper, we focus on one of the variants of lot-sizing problems mentioned in Jans and Degraeve (2008), namely the multi-product single-resource discrete lot-sizing and scheduling problem or DLSP. As defined in Fleischmann (1990) and Jans and Degraeve (2008), several key assumptions are used in the DLSP to model the production planning problem:

- A set of products is to be produced on a single capacitated production resource.
- A finite time horizon subdivided into discrete periods is used to plan production.
- Demand for products is time-varying (i.e. dynamic) and deterministically known.
- At most one product can be produced per period (small bucket model) and the facility processes either one product at full capacity or is completely idle (discrete or all-or-nothing production policy).

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- Costs to be minimized are the inventory holding costs and the changeover costs.

In the DLSP, it is assumed that a changeover between two production runs for different products results in a changeover cost and/or a changeover time. Changeover costs and times can depend either on the next product only (sequence-independent case) or on the sequence of products (sequence-dependent case). Significant changeover times which consume scarce production capacity tend to further complicate the problem. We consider here the DLSP with sequence-dependent changeover costs and times (denoted DLSPSD in what follows) and assume that the changeover times are expressed as integer numbers of planning periods and satisfy the triangular inequality.

Sequence-dependent changeover costs and times are mentioned in Jans and Degraeve (2008) as one of the relevant operational aspects to be incorporated into lot-sizing models. Moreover, a significant number of real-life lot-sizing problems involving sequence-dependent changeover costs and times have been recently reported in the academic literature: see among others (Dastidar & Nagi, 2005) for an injection moulding process, Silva and Magalhaes (2006) for a textile fibre industry or (Ferreira, Clark, Almada-Lobo, & Morabito, 2012) for soft drink production.

A wide variety of solution techniques from the Operations Research field have been proposed to solve lot-sizing problems: the reader is referred to (Buschkühl, Sahling, Helber, & Tempelmeier, 2010; Jans & Degraeve, 2007) for recent reviews on the corresponding literature. The present paper belongs to the line of research dealing with exact solution approaches, i.e. aiming at providing guaranteed optimal solutions for the problem. A large amount of existing solution techniques in this area consists in formulating the problem as a mixed-integer linear program (MILP) and in relying on a Branch & Bound type procedure to solve the obtained MILP. However the efficiency of such a procedure strongly depends on the quality of the lower bounds used to evaluate the nodes of the search tree. Much research has been devoted to the polyhedral study of lot-sizing problems and tight MILP formulations are now available for many variants of lot-sizing problems: see e.g. (Pochet & Wolsey, 2006) for a general overview of the related literature and (Belvaux & Wolsey, 2001; Eppen & Martin, 1987; Gicquel, Minoux, & Dallery, 2009; van Eijl & van Hoesel, 1997) for contributions focusing specifically on the DLSP.

Nevertheless, even if substantial improvements of the lower bounds can be obtained by using these MILP strengthening techniques, there are still cases where the obtained linear reformulation of the DLSPSD provides lower bounds of rather weak quality (see e.g. the numerical results reported in Gicquel et al. (2009)). These difficulties thus motivate the study of more powerful formulations for the problem. One such possibility consists in using a semidefinite reformulation of the problem rather than the standard linear reformulation used in MILP-based solution approaches.

Semidefinite programming (SDP) is a recent area of mathematical programming which can broadly be described as the extension of linear programming from the space of real vectors to the space of symmetric matrices: variables of the optimization problem are semidefinite positive matrices instead of positive real vectors. Since the seminal papers (Goemans & Williamson, 1995; Lovàsz & Schrijver, 1991) were published, semidefinite programming and its use to solve quadratic optimization problems have attracted a keen interest among researchers. Thanks to this, there is now a rather good knowledge on how to efficiently reformulate a quadratic optimization problem into a semidefinite program (see e.g. Roupin, 2004). Semidefinite programming has thus proved succesful at providing tight bounds for some well known quadratic binary problems such as the quadratic knapsack problem or the quadratic assignment problem (see e.g. Helmberg, Rendl, & Weismantel, 2000; Povh & Rendl, 2009; Zhao, Kharisch, Rendl, & Wolkovicz, 1998). However, applications of semidefinite programming in the field of industrial production management are still scarce (see Anjos, Kennings, & Vannelli, 2005; Mhanna & Jabr, 2012; Skutella, 2001 for noticeable exceptions) and to the best of our knowledge, there is no previous attempt at using semidefinite programming to solve lot-sizing problems. The purpose of the present paper is thus to provide a first assessment of the potential of semidefinite programming based approaches to solve discrete lot-sizing problems.

The main contributions of the present paper are thus threefold. First we introduce a new quadratically constrained quadratic binary programming formulation for the DLSPSD. Second, we propose to compute lower bounds for the DLSPSD using a semidefinite reformulation of the problem rather than a standard linear reformulation. Finally we present a cutting-plane generation algorithm based on a semidefinite programming solver to tighten the initial semidefinite relaxation. The results of the computational experiments carried out on small to medium-size instances show that the proposed approach provides lower bounds of significantly improved quality as compared to those provided by the best previously published linear reformulations, especially for the instances featuring a product family structure. Furthermore, for a high proportion of the small-size instances, the residual gap between the semidefinite relaxation and the optimal integer solution value is entirely closed so that there would be no need to resort to a Branch & Bound procedure to obtain the optimal integer solution. However, due to the limitations of available state-of-the art semidefinite programming solvers, these tight lower bounds are obtained at the expense of significant computation times.

The paper is organized as follows. We introduce in Section 2 a quadratically constrained quadratic binary programming (QCQBP) formulation for the DLSPSD. We then explain in Section 3 how this QCQBP can be reformulated as a semidefinite program and how lower bounds can be obtained for the DLSPSD by semidefinite relaxation. To achieve this, we not only exploit reformulation and strengthening techniques recently proposed in the SDP literature for generic (0–1) quadratic binary problems but also use problem-specific information such as the polyhedral representation of single-product discrete lot-sizing problems. Section 4 is devoted to the description of the valid inequalities used to strengthen the initial semidefinite relaxation of the problem and to the presentation of the cutting-plane generation algorithm implemented to add these valid inequalities iteratively into the initial formulation. Some computational results involving a comparison with the best previously published MILP strengthening techniques are then presented in Section 5.

2. QCQBP formulation of the DLSPSD

We first discuss a new formulation of the DLSPSD as a quadratically constrained quadratic binary (QCQBP) program. The sequence-dependent nature of the changeover costs namely leads to the introduction of a series of quadratic terms in the objective function. Moreover, inequalities involving quadratic terms are needed to ensure that the positive changeover times between different production runs for different products are respected.

2.1. Initial QCQBP formulation

We wish to plan production for a set of products denoted $p = 1, \ldots, P$ to be processed on a single production machine over a planning horizon involving $t = 1, \ldots, T$ periods. Product $p = 0$ represents the idle state of the machine and period $t = 0$ is used to describe the initial state of the production system.

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