European Journal of Operational Research 237 (2014) 628–633

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Stochastic preference analysis in numerical preference relations

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ARTICLE INFO

Article history:
Received 4 May 2013
Accepted 30 January 2014
Available online 2 March 2014

Keywords:
Decision analysis
Preference relation
Stochastic methods
Simulation

ABSTRACT

Numerical preference relations (NPRs) consisting of numerical judgments can be considered as a general form of the existing preference relations, such as multiplicative preference relations (MPRs), fuzzy preference relations (FPRs), interval MPRs (IV-MPRs) and interval FPRs (IV-FPRs). On the basis of NPRs, we develop a stochastic preference analysis (SPA) method to aid the decision makers (DMs) in decision making. The numerical judgments in NPRs can also be characterized by different probability distributions in accordance with practice. By exploring the judgment space of NPRs, SPA produces several outcomes including the rank acceptability index, the expected priority vector, the expected rank and the confidence factor. The outcomes are obtained by Monte Carlo simulation with at least 95% confidence degree. Based on the outcomes, the DMs can choose some of them which they find most useful to make reliable decisions.

1. Introduction

In decision making, preference relations are used to collect the preferences provided by decision makers (DMs). Then by prioritization methods, priorities can be obtained to rank the compared objectives in the preference relations. If input data in a preference relation are numerical values, then we can call it a numerical preference relation (NPR). NPRs have wide applications in multi-criteria decision making (Büyüközkan & Çifçi, 2011; Chiclana, Herrera, & Herrera-Viedma, 1998; Ho, Xu, & Dey, 2010; Saaty, 2008; Vaidya & Kumar, 2006; Zhu & Xu, 2014), where multiplicative preference relations (MPRs) (Saaty, 1980) and fuzzy preference relations (FPRs) (Orlovsky, 1978) are two basic ones.

Based on a 1–9 scale, the DMs provide deterministic point estimates as judgments to represent their preferences over paired comparisons of objectives to construct MPRs. Based on a 0.1–0.9 scale, FPRs can be constructed in a similar way. When considering uncertainties, many other possible representations of judgments can be used according to practice, such as intervals and random variables. To analyze the representations of judgments in multi-criteria decision making methods, Hahn (2003) gave systematic investigations shown in Fig. 1. With respect to the typology in Fig. 1, Type A and Type B indicate that errors in judgments are assumed to be nonexistent. When judgments are made with some degrees of error, they can be represented by intervals as in Type C. In Type D, the judgments are considered as a realization of stochastic phenomena.

With respect to NPRs, Types A and B correspond with FPRs and MPRs, respectively. Type C corresponds with interval MPRs (IV-MPRs) (Saaty & Vargas, 1987) and interval FPRs (IV-FPRs) (Xu, 2004b). It is clear that MPRs and FPRs are two special cases of IV-MPRs and IV-FPRs respectively. If some judgment(s) in NPRs are indicated by stochastic variable(s), then the NPRs are with probability interpretations.

The prioritization methods for NPRs in Types A, B and C are deterministic procedures, but Type D is with stochastic procedures. Hahn (2003) argued that the prioritization methods with deterministic procedures are special cases of their stochastic counterparts. So the stochastic methods represent a generalization of the deterministic ones. Many valuable prioritization methods with deterministic procedures have been developed, such as the classic eigenvector method (Saaty, 1977), the goal programming methods (Choo & Wedley, 2004; Fan, Ma, Jiang, Sun, & Ma, 2006; Xu, 2004a; Xu & Chen, 2008), the least deviation method (Xu & Da, 2005), the chi-square method (Wang, Fan, & Hua, 2007), the least-square method (Gong, 2008) and the fuzzy linear programming method (Zhu & Xu, 2014).

For the prioritization methods using stochastic procedures associated with stochastic variables that indicate judgments in Type D, Hauser and Tadikamalla (1996), Rosenbloom (1997), Levary and Wan (1998, 1999), and Banuelas and Antony (2006) developed some simulation methods based on MPRs in analytical hierarchy process, where probability distributions are used to describe the judgments.

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http://dx.doi.org/10.1016/j.ejor.2014.01.068
0377-2217/

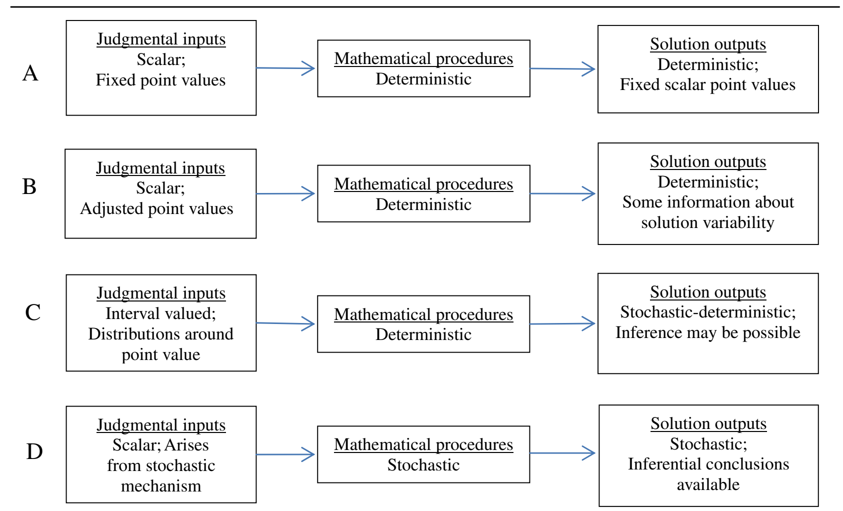

Fig. 1. A typology of multi-criteria decision making methods.

From the discussion above, we find that the prioritization methods for NPRs depend on the representations of judgments. However, considering the common characteristic of NPRs that consist of numerical judgments, is there a general prioritization method? In this paper, we develop stochastic preference analysis (SPA) as a new prioritization method for NPRs from a stochastic point of view, where the NPRs can be with different representations of numerical judgments. The results produced by this method are with probability interpretations. The structure of this paper is as follows. Section 2 reviews some basic concepts, then develops the concept of NPRs. In Section 3, we develop SPA. Section 4 gives two numerical examples. Section 5 provides a necessary discussion. The paper ends with some conclusions in Section 6.

2. Numerical preference relations

In this section, we introduce the basic forms of numerical preference relations (NPRs), that are multiplicative preference relations (MPRs) and fuzzy preference relations (FPRs). Some consistency measures are also introduced, which are used to develop stochastic preference analysis (SPA) in the next section.

For a set of objectives $X = \{x_1, x_2, \ldots, x_n\}$, the definition of MPRs can be stated as follows.

Definition 1 Saaty, 1980. A MPR on X is represented by $A \in X \times X$, $A = (a_{ij})_{n\times n}$, where a_{ij} indicates the degree that x_i is preferred to x_j, $a_{ij} \in \{1/9, 1/8, \ldots, 1/2, 1, 2, \ldots, 9\}$, and $a_{ij}a_{ji} = 1$.

In $A = (a_{ij})_{n\times n}$, $a_{ij} = 1$ indicates indifference between x_i and x_j; $a_{ij} > 1$ indicates that x_i is preferred to x_j; $a_{ij} < 1$ indicates that x_j is preferred to x_i.

Saaty (1980) defined the consistency on A as perfect if

$$a_{ij} = \frac{\omega_i}{\omega_j}, \quad \forall\ i,j = 1,2,\ldots,n \tag{1}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the priority vector of A, satisfying $\sum_{i=1}^{n}\omega_i = 1$, $\omega_i > 0$, $i = 1, 2, \ldots, n$.

Definition 2 Orlovsky, 1978. A FPR on X is represented by $R \in X \times X$, $R = (r_{ij})_{n\times n}$ is characterized by a membership function, $\mu_R: X \times X \to [0, 1]$, where $r_{ij} = \mu_R(x_i, x_j)$ is interpreted as the preference degree or the intensity of the x_i over x_j, and $r_{ij} + r_{ji} = 1$.

In $R = (r_{ij})_{n\times n}$, $r_{ij} = 0.5$ indicates indifference between x_i and x_j ($x_i \sim x_j$); $r_{ij} = 1$ indicates that x_i is absolutely preferred to x_j; $r_{ij} > 0.5$ indicates that x_i is preferred to x_j ($x_i \succ x_j$).

Tanino (1984) defined that R is multiplicative consistent if

$$r_{ij} = \frac{\omega_i}{\omega_i + \omega_j}, \quad \forall\ i,j = 1,2,\ldots,n \tag{2}$$

or R is additive consistent if

$$r_{ij} = 0.5(\omega_i - \omega_j + 1), \quad \forall\ i,j = 1,2,\ldots,n \tag{3}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the priority vector of R, satisfying $\sum_{i=1}^{n}\omega_i = 1$, $\omega_i > 0$, $i = 1, 2, \ldots, n$.

Based on the definitions of MPRs and FPRs, interval MPRs (IV-MPRs) developed by Saaty and Vargas (1987), and interval FPRs (IV-FPRs) developed by Xu and Chen (2008) can be restated respectively as follows.

Definition 3. An IV-MPR can be represented by $\widetilde{A} = (\tilde{a}_{ij})_{n\times n}$, where $\tilde{a}_{ij} = [a_{ij}^L, a_{ij}^U]$, $a_{ij}^L = 1/a_{ji}^U$ and $a_{ij}^U = 1/a_{ji}^L$ for all $i, j = 1, 2, \ldots, n$.

Definition 4. An IV-FPR can be represented by $\widetilde{R} = (\tilde{r}_{ij})_{n\times n}$, where $\tilde{r}_{ij} = [\tilde{r}_{ij}^L, \tilde{r}_{ij}^U]$, $\tilde{r}_{ij}^L = 1 - \tilde{r}_{ji}^U$ and $\tilde{r}_{ij}^U = 1 - \tilde{r}_{ji}^L$ for all $i, j = 1, 2, \ldots, n$.

Using numerical values to represent judgments, NPRs encompass MPRs, FPRs, IV-MPRs and IV-FPRs as special cases. We give a definition as follows.

Definition 5. A NPR on X is represented by $Z \in X \times X$, $Z = (z_{ij})_{n\times n}$, where z_{ij} indicates the degree(s) that x_i is preferred to x_j in the form of numerical value(s), such as a single numerical value, an interval numerical value, or several possible numerical values.

Generally, for one NPR, the numerical values should be given based on the same scale, such as the 1–9 scale associated with MPRs or the 0.1–0.9 scale associated with FPRs. So NPRs can be MPRs, FPRs, IV-MPRs or IV-FPRs etc., which are also the preference relations we take into account in this paper. Moreover, hesitant fuzzy preference relations (Zhu & Xu, 2013) and hesitant multiplicative preference relations (Xia & Xu, 2013) are also special cases of NPRs.

3. Stochastic preference analysis

SPA is a decision support method to aid the decision makers (DMs) to make informed decisions. It is motivated by a stochastic multi-criteria acceptability analysis (SMAA) method, originally introduced by Lahdelma, Hokkanen, & Salminen, 1998. SMAA is a family of methods for supporting multi-criteria decision making