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Innovative Applications of O.R.

Upper and lower bounds for the sales force deployment problem with explicit contiguity constraints

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ABSTRACT

The sales force deployment problem arises in many selling organizations. This complex planning problem involves the concurrent resolution of four interrelated subproblems: sizing of the sales force, sales representatives locations, sales territory alignment, and sales resource allocation. The objective is to maximize the total profit. For this, a well-known and accepted concave sales response function is used. Unfortunately, literature is lacking approaches that provide valid upper bounds. Therefore, we propose a model formulation with an infinite number of binary variables. The linear relaxation is solved by column generation where the variables with maximum reduced costs are obtained analytically. For the optimal objective function value of the linear relaxation an upper bound is provided. To obtain a very tight gap for the objective function value of the optimal integer solution we introduce a Branch-and-Price approach. Moreover, we propose explicit contiguity constraints based on flow variables. In a series of computational studies we consider instances which may occur in the pharmaceutical industry. The largest instance comprises 50 potential locations and more than 500 sales coverage units. We are able to solve this instance in 1273 seconds with a gap of less than 0.01%. A comparison with Drexl and Haase (1999) shows that we are able to halve the solution gap due to tight upper bounds provided by the column generation procedure.

1. Introduction

In many selling organizations, sales force deployment is an important tool by which sales management improves profit remarkably (Cravens & LaForge, 1983; Lodish, Curtis, Ness, & Simpson, 1988; Capron & Hulland, 1999 & Zoltners & Lorimer, 2000). It involves the concurrent resolution of four interrelated subproblems:

- sizing of the sales force,
- locations of the sales representatives,
- sales territory alignment, and
- sales resource allocation.

All these subproblems have to be solved simultaneously so that the total profit contribution will be maximized (Drexl & Haase, 1999). The subproblems can be briefly described as follows: Consider a large geographical market area that is partitioned into a set of so-called sales coverage units (SCUs). *Sizing of the sales force* implies selecting the appropriate number of sales representatives required to penetrate the market area. A sales territory and a location are unequivocally assigned to each sales representative. Thus, by sales force sizing we also decide on the number of sales territories and locations. The *salesperson location* aspect of the problem involves determining a SCU for a sales representative to be located in. *Sales territory alignment* may be viewed as the partitioning problem of grouping the SCUs around the locations of the sales representatives into larger geographic areas called "sales territories". *Sales resource allocation* refers to the problem of allocating a sales representative's time to the assigned SCUs. Note, sales force deployment is an aggregate planning problem. The usual planning horizon is one year. So, decisions as how often (e.g., every week, once a month), which days and at what time a customer is to be visited (and the sequence of customers) are the subject of subsequent planning stages (Golalikhani & Karwan, 2013).

Comprehensive reviews of related contributions are given by Drexl and Haase (1999) and Howick and Pidd (1990). Although there is a long tradition of sales force deployment modeling (particularly sales territory alignment) as shown by Lodish (1975), Zoltners (1976), Rangaswamy, Sinah, and Zoltners (1990) and Albers (1996) for example, we see that this field of research is still very popular (Segura-Ramiro, Ríos-Mercado, Álvarez Socarrás, &

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de Alba Romenus, 2007; Mantrala et al., 2010 & Lee & Yang, 2013) – especially if we consider the practical implementation of approaches (Zolterns & Sinah, 2005). To our novel approach the following work is of particular interest and is therefore discussed in more detail. Skiera and Albers (1998) formulate a model that addresses both the sales territory alignment and the sales resource allocation problems. They propose sales response functions of any given concave form at the level of SCUs. Sales are modeled as a function of selling time, that includes calling time as well as travel time, assuming a constant ratio of travel time to calling time. They consider a resource allocation model and a territory alignment model simultaneously. For the solution a so-called backward deletion procedure is considered. If desired, contiguous sales territories can be constructed by the heuristic. However, explicit contiguity constraints – that is, the constraints used in the model guarantee contiguous sales territories – are not considered. In Drexl and Haase (1999) all four subproblems are covered in order to maximize profit. For the solution of the model they present approximation methods capable of solving large-scale, real-world instances. The methods that provide lower bounds for the optimal objective function value are compared to upper bounds. On average the solution gap, i.e., the difference between upper and lower bounds with respect to the upper bound, is about 3%. Furthermore, they show how the methods can be used to analyze various problem settings of practical relevance.

We present a novel approach to compute tight upper and lower bounds for the sales force deployment problem based on an innovative mathematical program that explicitly accounts for contiguous sales territories. Although contiguous sales territories are compulsory in applications, explicit contiguity constraints are rare so far. Zoltners and Sinah (1983) propose a mixed integer formulation to account for contiguous sales territories. However, this approach is not explicit in terms of contiguity and as a consequence contiguity is not guaranteed (Shirabe, 2005). The contiguity constraints in Drexl and Haase (1999) grow exponentially with the number of spatial units (King, Jacobson, Sewell, & Cho, 2012). In contrast, our approach explicitly accounts for contiguity due to a set of contiguity constraints based on continuous flow variables. By this, we are able to significantly reduce the number of constraints. Further, our model is characterized by an infinite number of binary variables where each variable is related to a point of selling time in a time interval (selling time variables). For its solution we propose a Branch-and-Price approach, i.e., at each node of a Branch-and-Bound tree we solve a master problem by column generation. The corresponding restricted master problem consists of the linear relaxation of an integer formulation with a reduced number of variables. Analytical solutions are introduced for the subproblem. We derive an upper bound for the optimal objective function value of the linear relaxation using dual information. Branching is done on two categories of binary variables. At first, we branch on the variables indicating where locations for sales representatives should be setup. Afterward, we consider assignment variables indicating to which location a sales coverage unit should be assigned. Moreover, we present an intelligible approach to determine an initial lower bound.

Managers might use mathematical sales force deployment approaches to generate good initial solutions and then "fine-tune" the solution by individual expertise (Zolterns & Sinah, 2005). Usually, "what-if" scenarios are employed to improve overall sales performance. Therefore, fast and valid solutions of the mathematical model are needed in order to perform such a "management-heuristic" efficiently (Pinals, 2001). Our computational studies show that especially for large practical instances the new approach provides an outstanding solution quality in reasonable time.

The organization of this paper is as follows: Section 2 describes the problem setting. In particular, we define important terms and explain assumptions that are implicitly involved in these definitions. Section 3 introduces a new mathematical model for sales force deployment. Section 4 provides a Branch-and-Price approach to this formulation to derive tight lower and upper bounds. Section 5 covers the results of computational studies. To stress the practical relevance, we consider a large application-oriented example within the health care and pharmaceutical industry in Germany. The summary and conclusions are provided in Section 6.

2. Problem description

Subsequently, we explain and discuss terms that are important for understanding the addressed problem. Additionally, we provide a problem statement.

- **Account**: A customer (company, self-employed persons, etc.) who is expected to buy products of the company that employs the sales representative.
- **Sales Coverage Unit (SCU)**: A sales coverage unit is considered here to be a relatively small geographical area. The choice and thus the size of the SCUs depend upon the specific application and on whether the required data can be obtained (at a reasonable cost). Counties, zip codes, and company trading areas are some examples of SCUs (see for instance Zoltners & Sinah (1983) and Churchill, Ford, & Walker (1993)). A SCU might contain several accounts.
- **Sales territory**: A sales territory is a geographic area that consists of a set of SCUs with a responsible sales representative. To be responsible means that the sales representative has to provide service for all (potential) accounts located in the corresponding sales territory (Darmon, 2002). For example, a sales representative who is working for a company that provides materials concerning dental surgery will be responsible for all dentists practicing in her sales territory. She is only allowed to sell products in SCUs that belong to her sales territory. Contiguity – that is, the SCUs of a sales territory are connected (Church & Murray, 2009) – is usually demanded by the management of a company for some organizational reasons.
- **Travel time, calling time, selling time**: In order to sell a product a sales representative has to do some time consuming activities. The time required to travel from a location to an account, from an account to another account, and back to the location is called travel time. Although travel time might be considered as routing time, we neglect this fact here and assume travel time is only affected by the distance between SCU and the location of the sales representative. This is reasonable, because sales force deployment is a strategic management issue while routing is more related to operational management. Presenting a product and performing contract negotiations are examples of calling activities. The associated time is denoted as calling time. Now, selling time is defined as the sum of calling time and travel time. We have no specific information about the number of calls to individual accounts because our sales response functions are based on the SCU level.[1] Thus, we assume that the sales response function represents sales with selling time optimally allocated across accounts. For each sales representative the total selling time is restricted. We assume that the calling time is a constant fraction of the selling time. At a first glance, this appears to be a more or less rough average consideration. However, Skiera and Albers (1998) show that this assumption holds if mild conditions are given. For further details see Skiera and Albers (2008).

[1] It is easy to consider sales response function on the account level by generating a (artificial) SCU for each account.