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Evaluating corporate bonds with complicated liability structures and bond provisions ☆

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ABSTRACT

This paper presents a general and numerically accurate lattice methodology to price risky corporate bonds. It can handle complex default boundaries, discrete payments, various asset sales assumptions, and early redemption provisions for which closed-form solutions are unavailable. Furthermore, it can price a portfolio of bonds that accounts for their complex interaction, whereas traditional approaches can only price each bond individually or a small portfolio of highly simplistic bonds. Because of the generality and accuracy of our method, it is used to investigate how credit spreads are influenced by the bond provisions and the change in a firm's liability structure due to bond repayments.

1. Introduction

A risk-free bond can be priced by simply summing all the discounted future cash flows, independently of other outstanding bonds of the same issuer. A risky bond in sharp contrast must be priced simultaneously with other outstanding bonds of the same issuer because the issuance and *each* repayment of a bond changes the financial status of the issuer and thus the likelihood of default of all the bonds, even more senior ones. This point is most serious for an issuer with multiple bonds outstanding and a complex liability structure. This complex interaction among the bonds due to their payment schedules and provisions makes pricing them beyond the reach of analytic approaches (see Lando, 2004).

Given a firm's liability structure, a credit model is needed to price risky bonds (Chiarella, Fanelli, & Musti, 2011; Onorato & Altman, 2005; Saunders, Xiouros, & Zenios, 2007; Westgaard & Van der Wijst, 2001). One of such models, the structural model, specifies the evolution of the firm's asset value and the conditions leading to default (see Merton, 1974), from which the change in a firm's liability structure follows naturally. The bond repayment financed by selling the firm's asset, for example, is modeled by a downward move in the firm's asset value. So structural models make explicit the connection between default and the firm's assets and liabilities. This paper will focus on structural models.

Merton (1974) assumes the firm's asset value follows a lognormal diffusion process and default can only occur at the single bond's maturity date when the firm's asset value cannot meet its payment obligations. Therefore, equities can be viewed as call options on the firm's asset and can be priced by the Black–Scholes formula (see Black & Scholes, 1973). Black and Cox (1976) develop the first-passage model, which assumes the firm issues only one bond and it defaults once the asset value hits an exogenous default boundary. The single-bond case is clearly too restricted for practical applications. Geske (1977) is the first to price a risky bond in the presence of other outstanding bonds. He considers a portfolio consisting of a senior bond with a maturity date of T_1 and a subordinated one with a later maturity date of T_2. Then he applies the compound-option framework to price both bonds. In summary, the analytical methods can only price each bond individually or a very small portfolio of highly simplistic bonds (see Ericsson & Reneby, 1998; Glasserman & Nouri, 2012). Generalizing them to more complicated liability structures remains elusive.

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Bond provisions such as restrictions on asset sales, exogenous default boundaries, seniorities of bonds, and early redemption (like put provisions) affect risky bond prices profoundly. We now go over each of them briefly. The value of a risky bond depends strongly on the assumptions regarding asset sales (see Lando, 2004). To protect the bond holders, bond provisions may prohibit equity holders from selling the firm's asset to finance bond repayments or dividend payouts. This no-asset-sales assumption is often needed for closed-form solutions (see Leland, 1994). But allowing asset sales is more common in the real world. To loosen the restriction on asset sales while keeping the problem analytically solvable, some papers adopt the proportional-asset-sales assumption, which allows the firm to sell a proportion of its asset (see Kim, Ramaswamy, & Sundaresan, 1993; Leland, 1994; Hilberink & Rogers, 2002). Besides the two aforementioned assumptions, Merton (1974) and Brennan and Schwartz (1978) assume the payout can be fully financed by selling the firm's asset. We call this third assumption the total-asset-sales assumption. This assumption significantly increases the difficulty to price the bonds, analytically or otherwise. This is because a *fixed* amount of the firm's asset is sold to finance the repayments, which is essentially the well-known problem faced by option pricing with fixed dividends (see Dai, 2009).

We now move onto exogenous default boundaries. The positive-net-worth covenant forces the firm into bankruptcy if its asset value hits an exogenous default boundary that depends on the firm's liability structure (see Brennan & Schwartz, 1978; Kim et al., 1993; Longstaff & Schwartz, 1995; Nielsen, Saá-Requejo, & Santa-Clara, 2001; Briys & De Varenne, 1997). Note that a complex liability structure entails a complex exogenous default boundary. We follow Leland (1994) in calling a bond with an exogenous default boundary a protected bond. The default boundary can also be determined endogenously based on assumptions on asset sales. For example, under the no-asset-sales and proportional-asset-sales assumptions, the firm defaults if the equity holders fail to raise enough equity capital to meet the bond payments (see Leland, 1994), whereas under the proportional-asset-sales and total-asset-sales assumptions, the firm defaults when the firm's asset is insufficient to cover the payments. Note that the default boundary for a protected bond is shaped by both the exogenous and the endogenous default boundaries. In contrast, the default boundary of a bond without protection from the positive-net-worth covenant is simply the endogenous default boundary. We follow Leland (1994) in calling this bond an unprotected bond.

Seniority refers to payment priority in the event of bankruptcy. When the issuer goes bankrupt, senior bonds are repaid before subordinated ones. But a subordinated bond may still affect the risk of the senior ones. This is because when a firm is allowed to sell its asset to finance the bond repayments, the repayment of a subordinated bond before the maturity of the senior ones increases the risk of the latter as the asset sale changes the financial status of the firm.

Finally, we discuss early redemptions. The putable provision provides some protection for the bond holders against the increase in interest rate, which reduces the bond value. Our paper will also show that it can provide some protection against the issuer's credit risk. This is because bond holders can exercise the putable right before the firm's financial status is weakened due to scheduled bond repayments.

In summary, real-world bond provisions and the complex interaction among bonds make pricing risky bonds infeasible, in most cases, for analytical approaches and challenging for numerical ones. To rectify the situation, this paper develops a general lattice methodology for pricing corporate bonds with complicated liability structures and bond provisions under the structural model. A lattice is a popular numerical method. It divides a certain time interval into n time steps and the pricing results converge to the theoretical price as $n \to \infty$ (see Duffie, 1996). However, some provisions such as exogenous default boundaries will cause naive implementations to experience price oscillations as Fig. 1 shows. To eliminate the oscillations, we incorporate the techniques of Dai and Lyuu (2010) to makes certain nodes or price levels on the lattice align with the exogenous default boundaries. In addition, the trinomial structure of Dai (2009) is used to handle the discontinuities in the firm's asset value resulting from asset sales. Backward induction then handles bond provisions such as seniority and embedded options.

With our proposed lattice method in place, the paper explores how credit spreads are influenced by the bond provisions and change in the firm's liability structure due to each bond repayments. Numerical results reveal that they greatly affect bond prices, sometimes in unexpected ways. Complex scenarios such as this are hard to analyze by the traditional approaches, but they pose no difficulties for our lattice. Finally, our methodology is flexible enough to make it applicable to other complex option-related problems such as real options (see Ho & Yi, 2004; Zmeškal, 2010).

Our paper is organized as follows. The model, lattice constructions, and the oscillation problem are introduced in Section 2. Section 3 describes how our lattice is constructed to cope with complicated liability structures and various bond provisions. Section 4 details how bond provisions are handled in backward induction. Section 5 analyzes the price behaviors of risky bonds for complicated liability structures and various bond provisions. Section 6 concludes.

2. Basic terms and preliminaries

2.1. The dynamics of the firm's asset value

Denote the firm's asset value at time t as V_t, whose dynamics follows the following process (see Merton, 1974),

$$dV_t = (rV_t - P)\,dt + \sigma V_t\,dz. \tag{1}$$

Above, r is the risk-free rate, P denotes the firm's payout financed by selling the firm's asset per annum, σ denotes the volatility, and dz is a standard Brownian motion (see Black & Scholes, 1973; Osborne, 1959). P can depend on V_t and t.

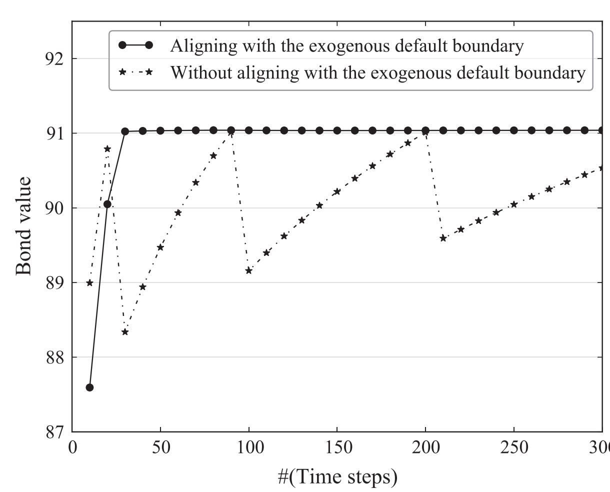

Fig. 1. The price oscillation phenomenon. The firm's asset value is assumed to follow the lognormal diffusion process. The firm's initial asset value is \$100, the risk-free interest rate is 1%, and the volatility of the asset value is 25%. The firm issues a zero-coupon bond with one-year maturity and face value \$95. The exogenous default boundary is set to \$90. The prices oscillate significantly if the lattice does not align with the exogenous boundary.