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Inventory games with permissible delay in payments

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ABSTRACT

Meca et al. (2004) studied a class of inventory games which arise when a group of retailers who observe demand for a common item decide to cooperate and make joint orders with the EOQ policy. In this paper, we extend their model to the situation where retailer's delay in payments is permitted by the supplier. We introduce the corresponding inventory game with permissible delay in payments, and prove that its core is nonempty. Then, a core allocation rule is proposed which can be reached through population monotonic allocation scheme. Under this allocation rule, the grand coalition is shown to be stable from a farsighted point of view.

1. Introduction

The inventory games were introduced by Meca, Timmer, García-Jurado, and Borm (2004). This class of games arises when a group of retailers who observe demand for a common item decide to cooperate and make joint orders. By placing joint orders, these retailers can reduce their purchase cost and transaction-related cost due to economies of scale and transaction cost economics (Schotanus, Telgen, & de Boer, 2010). This kind of horizontal cooperation is becoming increasingly popular in the economic world since the supply chain arena has undergone radical changes in recent years with increasing emphasis on cooperation and information sharing. A very recent survey on cooperation among supply chain agents can be seen in Nagarajan and Sošić (2008), Meca and Timmer (2008).

In this paper, we extend the inventory cost games studied in Meca et al. (2004) to the situation with permissible delay in payments. Permissible delay in payments (trade credit) is very common in supply chain transactions. For retailers, the benefits that they can obtain from permissible delay in payments are obvious. For example, retailers can take advantage of permissible delay in payments as a source of financing when they are short of cash. Especially, small and medium-sized firms who have difficult access to external financing may largely rely on trade credit as a main source of short-term funds. Moreover, once delay in payments is permitted for retailers, the amount of time that retailers' capital invests in stock is reduced, which accordingly leads to a reduction in retailers' stock-holding cost. For suppliers, permissible delay in payments can promote their sales and reduce their on-hand stock level. According to Rajan and Zingales (1995), the volume of trade credit in aggregate represents 17.8% of total assets for US firms, 22% for UK firms, and more than 25% for countries such as Germany, France and Italy. Therefore, in today's business transactions, trade credit is one of the most important source of short-term external financing for firms in a broad range of industries and economies (Fisman, 2001).

Beranek (1967) emphasized the importance of paying attention to credit terms when making lot sizing decisions and gave examples to illustrate that ignoring financial considerations will lead to an infeasible stocking policy. Since then, there appeared extensive literatures on trade credit and inventory policy. Goyal (1985) suggested a mathematical model for obtaining the economic order quantity under permissible delay in payments. Aggarwal and Jaggi (1995) considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Jamal, Sarker, and Wang (1997) extended this issue with allowable shortage. However, Jamal et al. (1997) did not provide an accurate and reliable procedure to find the optimal solutions. Chung and Huang (2009) extended Goyal's model by considering allowable shortage and presented a theorem to determine the optimal order quantity. Teng (2002) amended Goyal's model by considering the difference between unit price and unit cost, and found that the economic replenishment interval and order quantity decrease when delay in payments is permitted in certain cases. Many other models dealing with trade credit problems can be seen in Liao, Tsai, and Su (2000), Song and Cai (2006), Chung and Huang (2003), Ouyang, Teng, and Chen (2006), Lee and Rhee (2011), and references therein.

Our paper is also related to the vast literature on applications of cooperative game theory in the area of inventory management. To the best of our knowledge, Hartman (1994) first applied

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cooperative game theory to study methods of allocating inventory costs and benefits. Since then, cooperative game theory is widely applied in this area. For example, the application of cooperative game theory to continuous-review inventory model with (Q, r) replenishment policy (Hartman & Dror, 1996), to economic order quantity models (see Meca, García-Jurado, and Borm (2003), Meca, Guardiola, and Toledo (2007) and Dror and Hartman (2007)), to economic lot-sizing models (see van den Heuvel, Borm, and Hamers (2007) and Guardiola, Meca, and Puerto (2008, 2009), among others), to newsvendor models (see Hartman, Dror, and Shaked (2000), Muller, Scarsini, and Shaked (2002), Hartman and Dror (2005), Slikker, Fransoo, and Wouters (2005), Özen, Fransoo, Norde, and Slikker (2008), Özen, Erikip, and Slikker (2012), Chen and Zhang (2009), and the many references therein), and also to inventory transportation systems (see Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2011a, 2013)). Dror and Hartman (2010), Fiestras-Janeiro, García-Jurado, Meca, and Mosquera (2011b) reviewed the applications of cooperative game theory to inventory management. These literature mostly focus on examining the existence of the core of the corresponding game, which analyzes the stability of the grand coalition from a myopic point of view. There are other literature applying the concept of Largest Consistent Set (LCS) to analyze coalition stability in supply chain from the farsighted perspective, for example, Granot and Sošić (2005), Sošić (2006), Nagarajan and Sošić (2007, 2009). In this paper, we will examine the core of inventory games with permissible delay in payments, as well as the farsighted stability of the grand coalition.

The contribution of our paper is threefold. First, we introduce the inventory game with permissible delay in payments and show that the core of this kind of game is non-empty. Second, we propose a cost allocation rule that is in the core which can be reached through population monotonic allocation schemes. Third, we examine the stability of grand coalition from a farsighted perspective, and show that grand coalition belongs to the largest consistent set, i.e., it is a farsighted stable outcome under the proposed cost allocation rule.

The rest of this paper is organized as follows. We start by introducing preliminaries on cooperative game theory in Section 2. In Section 3 we introduce the inventory model with permissible delay in payments under independent and cooperative purchase respectively. Section 4 presents the inventory game with permissible delay in payments. We offer concluding remarks in Section 5.

2. Preliminaries on cooperative game theory

A transferable utility game (a TU-game) is a pair (N, c), where N is a finite set of players and c is the characteristic function defined from the family of subsets of N with $c(\varnothing) = 0$. For any given $S \subseteq N, c(S)$ is defined as the minimal total cost that the members of S can achieve when they cooperate. A TU-game is said to be subadditive if $c(S \cup R) \leqslant c(S) + c(R)$ for each $S, R \subset N$ such that $S \cap R = \varnothing$. Clearly, there exists an incentive to cooperate in subadditive games with transferable utilities.

An allocation is a function x from N to $\mathbb{R}$ that assigns an amount of cost to each member of N. For each $x = (x_1, \ldots, x_n) \in \mathbb{R}^N, x_j$ is the cost allocated to retailer j, and $x(S)$ is the total amount allocated to coalition $S \subset N$ where $x(S) = \sum_{j \in S} x_j$. An allocation $x \in \mathbb{R}^N$ is efficient if $x(N) = c(N)$, and is individually rational if each player is charged less by joining the coalition than working alone, that is, $x_i \leqslant c(\{i\})$ for each $i \in N$. An imputation is an efficient and individually rational allocation. The set of all imputation of a TU-game (N, c) is denoted by

$I(N, c) = \{x \in \mathbb{R}^N | x(N) = c(N) \text{ and } x_i \leqslant c(\{i\}) \text{ for each } i \in N\}$

2.1. The core

The core of a TU-game (N, c) is the subset of all imputations $x \in I(N, c)$ that no coalition has an incentive to split off from the grand coalition and form subcoalitions, i.e., $x(S) \leqslant c(S)$. Allocations belonging to the core will be called core allocations in the rest of this paper. We denote the set of core allocations for the TU-game (N, c) by $Core(c)$, that is,

$$Core(c) = \left\{ x \in \mathbb{R}^N | \sum_{j \in N} x_j = c(N), \sum_{j \in S} x_j \leqslant c(S) \text{ for every } S \subset N,\ S \neq \varnothing \right\}$$

A cost game (N, c) has a non-empty core if and only if it is balanced; it is a totally balanced game if the core of every subgame is non-empty.

A population monotonic allocation scheme (PMAS), was introduced by Sprumont (1990) and defined as follows. For a TU-game (N, c), a vector $y^S \in \mathbb{R}^S$ is a PMAS if and only if it satisfies the following two conditions. First, for all $S \subset N, S \neq \varnothing, y^S(S) = c(S)$. Second, for all $S \subset R \subset N, i \in S, y_i^S \geqslant y_i^R$. It follows from Sprumont (1990) that each cost game with PMAS is totally balanced. An allocation x for TU-game (N, c) can be reached through a PMAS if there exists a PMAS $(y^S)_{\varnothing \neq S \subset N}$ such that $y_i^N = x_i$ for all $i \in N$.

As Chwe (1994) points out, membership in the core provides a kind of stability from myopic view. The idea behind the core is that if a subset of players can benefit by defecting from the grand coalition with one step, then the grand coalition is considered to be unstable. However, it precludes the possibility that an initial defection may trigger a sequence of further moves, which eventually can lead to an outcome wherein the players who initiated the deviations would receive higher cost than that they would obtain in the grand coalition. Therefore, farsighted players may not choose to defect in the first place, and thus the grand coalition, which appeared unstable from a myopic view, may actually be stable from a farsighted point of view. A new solution concept, the largest consistent set (LCS), which allows players to look ahead and consider possible further deviations, was introduced by Chwe (1994). Basically, the largest consistent set approaches stability analysis from a farsighted perspective, i.e., considers the effect of externalities and allows players to consider multiple possible further deviations, while the core approaches stability analysis from a myopic perspective, i.e., considers only one step deviation. In this paper, we will analyze coalition stability using both the core and the largest consistent set. The definition of the largest consistent set is below.

2.2. The largest consistent set

By $\mathscr{L}$ we denote coalition structures where $\mathscr{L}$ is a partition of the player set $N = \{1, 2, \ldots, n\}$, (i.e., $\mathscr{L} = \{L_1, L_2, \ldots, L_m\}$ such that $\cup_{i=1}^m L_i = N$ and $L_i \cap L_j = \varnothing, i \neq j$). For two coalition structures $\mathscr{L}_1, \mathscr{L}_2$, we say that play i strongly prefers coalition structure $\mathscr{L}_2$ to $\mathscr{L}_1$, i.e., $\mathscr{L}_1 \prec_i \mathscr{L}_2$ if the cost allocated to him/her under $\mathscr{L}_2$ is strictly lower than under $\mathscr{L}_1$. In other words, $\mathscr{L}_1 \prec_i \mathscr{L}_2 \iff x_i^{\mathscr{L}_2} < x_i^{\mathscr{L}_1}$, where $x_i^{\mathscr{L}}$ denotes player i's cost under coalition structure $\mathscr{L}$. For coalition $S, \mathscr{L}_1 \prec_S \mathscr{L}_2$, if $\mathscr{L}_1 \prec_i \mathscr{L}_2$ for all $i \in S$. By $\rightharpoonup_S$ we denote the following relation: $\mathscr{L}_1 \rightharpoonup_S \mathscr{L}_2$ if the coalition structure $\mathscr{L}_2$ is obtained when S deviates from coalition structure $\mathscr{L}_1$. We say that $\mathscr{L}_1$ is directly dominated by $\mathscr{L}_2$, i.e., $\mathscr{L}_1 < \mathscr{L}_2$ if there exists a coalition S such that $\mathscr{L}_1 \rightharpoonup_S \mathscr{L}_2$ and $\mathscr{L}_1 \prec_S \mathscr{L}_2$. We say that $\mathscr{L}_1$ is indirectly dominated by $\mathscr{L}_m$, i.e., $\mathscr{L}_1 \ll \mathscr{L}_m$ if there exist $\mathscr{L}_1, \mathscr{L}_2, \ldots, \mathscr{L}_m$ and $S_1, S_2, \ldots, S_m$ such that $\mathscr{L}_i \rightharpoonup_{S_i} \mathscr{L}_{i+1}$ and $\mathscr{L}_i \prec_{S_i} \mathscr{L}_m$ for $i = 1, 2, \ldots, m-1$.

A set $\mathcal{Y}$ is called consistent if the following condition holds: $\mathscr{L} \in \mathcal{Y}$ if and only if for all $\widehat{\mathscr{L}}, C$ such that $\mathscr{L} \rightharpoonup_C \widehat{\mathscr{L}}$ there exists $\mathcal{B} \in \mathcal{Y}$ where $\widehat{\mathscr{L}} = \mathcal{B}$ or $\widehat{\mathscr{L}} \ll \mathcal{B}$ such that $\mathscr{L} \nprec_C \mathcal{B}$. Chwe (1994) shows that although there can be many consistent sets, there uniquely exists a *Largest Consistent Set*, which contains all other consistent sets. The largest consistent set has the merit of "ruling out with confidence", that is, if an outcome is not in the largest consis-