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Discrete Optimization

Relations, models and a memetic approach for three degree-dependent spanning tree problems

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ABSTRACT

In this paper we take into account three different spanning tree problems with degree-dependent objective functions. The main application of these problems is in the field of optical network design. In particular, we propose the classical Minimum Leaves Spanning Tree problem as a relevant problem in this field and show its relations with the Minimum Branch Vertices and the Minimum Degree Sum Problems. We present a unified memetic algorithm for the three problems and show its effectiveness on a wide range of test instances.

1. Introduction

Spanning tree problems with degree-related objective functions or constraints are widely studied in the field of network design. In such problems, we generally look for spanning trees that optimize or respect certain properties related to the degree of the vertices, in order to model cost factors or restrictions deriving from the underlying real-world applications.

In the *Minimum Branch Vertices Problem* (or MBV), we look for the spanning tree with the minimum number of vertices (called *branch* vertices) with a degree higher than two.

This problem has great relevance, for instance, in the context of multicast on optical networks. On such networks, the optical signal can be split and therefore sent from a source to multiple destinations by using an appropriate network device (*switch*). Multicast communications can therefore be performed through a spanning tree of the network (light-tree), by placing a switch on each branch vertex. For several reasons (such as budget constraints or signal quality preservation, among others) it can be important to determine the spanning tree which requires the minimum number of switches, that is, the optimal solution for the MBV problem.

Indeed, many switch devices can only duplicate laser beams; therefore, the actual number of devices to be located on a branch vertex is related to the degree of the node. For this reason, the problem of minimizing the degree sum of the branch vertices of any spanning tree of the network (*Minimum Degree Sum Problem* or MDS) has been proposed in the literature.

However, as can be noted from Fig. 1, if $\delta(u, T)$ is the degree of a branch node that is used to propagate information, the exact number of required devices is $\delta(u, T) - 2$. More in general, in this context, the optimization problem consists in minimizing the degree sum of the branch vertices less the cardinality of the set of branch vertices multiplied by two. In this paper we show for the first time that this problem, which models the considered underlying application more accurately than MDS, is equivalent to the well known *Minimum Leaves Problem* (ML), i.e. the problem of finding the spanning tree with the minimum number of degree-1 vertices. This will be proved in Section 2.

The aim of this paper is therefore to propose ML as a relevant problem in the field of optical network design and to show that it is closely related to MDS and MBV, by demonstrating some theoretical properties linking their three objective functions. These objectives are also pursued by presenting a unified memetic algorithm that makes use of a single set of rules to perform crossover, mutation and local search operations for the three problems. An extensive experimental analysis proves the effectivity of the proposed approach.

MBV has been first introduced in Gargano, Hell, Stacho, and Vaccaro (2002) where the problem was shown to be $\mathcal{NP}$-Hard. In Carrabs, Cerulli, Gaudioso, and Gentili (in press) the authors present four different mathematical formulations and compare the results of different relaxations, solving the lagrangian dual by means of a standard subgradient method and an ad hoc finite ascent algorithm. In Gargano and Hammar (2003) and Gargano, Hammar, Hell, Stacho, and Vaccaro (2004), the authors give

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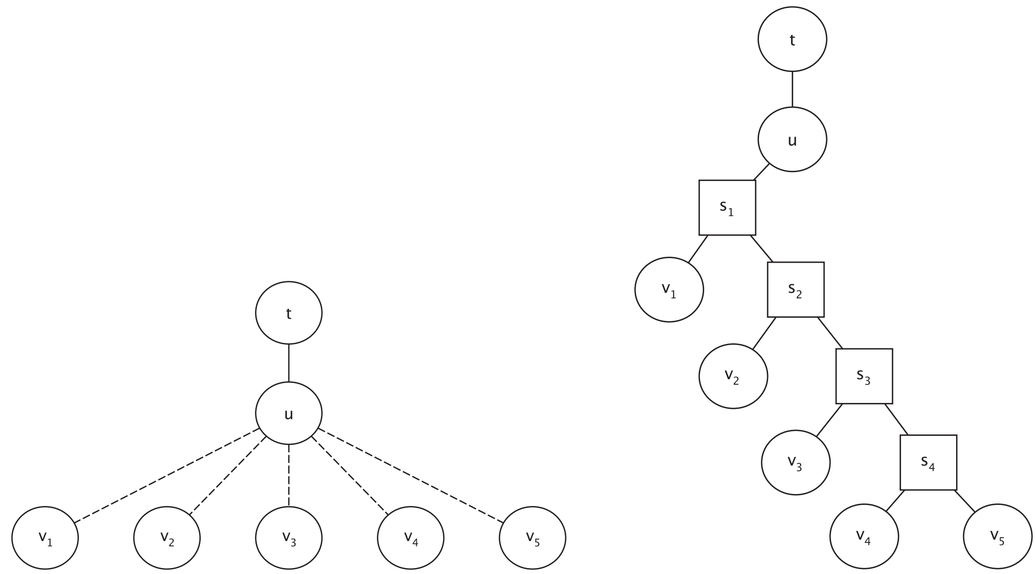

Fig. 1. Example subtree. Switches $s_1,\ldots,s_4$ need to be located on node u to transmit information from t to $v_1,\ldots,v_5$.

conditions for the existence of *spanning spiders* (i.e. spanning tree containing at most one branch vertex) and, more in general, spanning trees with a bounded number of branch vertices. Another problem related to MBV is the *degree-constrained minimum spanning tree*. Given an edge-weighted graph $G = (V, E)$ and a value $b(i) \geqslant 1$, $\forall i \in V$, the aim is to find a spanning tree with minimum weight such that the degree of each node i is bounded by $b(i)$. In Ribeiro and de Souza (2002), the authors solve the problem by presenting a VNS metaheuristic that embeds a Variable Neighborhood Descent (VND) strategy for local search. An improvement of this algorithm is described in de Souza and Martins (2008). The new algorithm uses guiding strategies based on the Second Order algorithm in the shaking phase and the Skewed approach to avoid degeneration into a multistart heuristic. A branch-and-cut method for solving the problem is discussed in Caccetta and Hill (2001). In Duhamel, Gouveia, Moura, and Souza (2011), the authors present a generalization of the problem which considers a non-linear stepwise cost function on every node. They present two linear programming formulations as well as a hybrid GRASP/VND metaheuristic embedding a Path Relinking strategy applied at the end of each GRASP iteration. MDS has been presented and analyzed in Cerulli, Gentili, and Iossa (2009), which also contains some mathematical formulations and heuristic procedures for both MBV and MDS. Other heuristic approaches for MBV and MDS have been recently proposed in Sundar, Singh, and Rossi (2012). The ML problem was proven to be $\mathcal{NP}$-Hard and hard to approximate in Lu and Ravi (1996). In Salamon and Wiener (2008), the authors introduce some approximation algorithms for the related problem of maximizing the number of internal nodes (of course, the two problems have identical optimal solutions). In Fernandes and Gouveia (1998), given an edge-weighted graph and a natural number $k \geqslant 1$, the authors study the problem of finding the minimum weight spanning tree with exactly k leaves. Two mathematical formulations derived from the minimum weight spanning tree problem as well as upper bounding and lower bounding schemes are presented.

The sequel of the paper is organized as follows. Section 2 contains the formal definition of the studied problems, and the demonstrations of several relations among them. Section 3 introduces some mathematical formulations for each of the three problems, and Section 4 describes the memetic algorithm that we propose to solve them. Section 5 includes the results of the extensive computational tests we performed to compare our memetic approach with the mathematical formulations solved by means of the CPLEX solver. Finally, Section 6 presents some final remarks.

2. Problems definitions and relations

2.1. Notation

Let $G = (V, E)$ be a connected undirected input graph, and $T = (V, E')$ be a subgraph of G. Let $V(T)_i \subseteq V$, $i \geqslant 0$ be the set of vertices with degree i in T. Moreover, define $V(T)_B = V \backslash \{V(T)_1 \cup V(T)_2\}$; that is, if T is a spanning tree, $V(T)_B$ is the set of its branch vertices.

Moreover, for each $j \in V$, let $\delta(j, T)$ be the degree of j in T. Note that if $j \in V(T)_i$, then $\delta(j, T) = i$. Finally, for each set of vertices $X \subseteq V$, let $\Delta(X, T)$ be the degree sum of the vertices of X in T $(\Delta(X,T) = \sum_{j\in X}\delta(j,T))$.

In order to better clarify the introduced notation, consider the tree T in Fig. 2. We have that $V(T)_1 = \{a, b, c, d, h\}$, $V(T)_2 = \{g\}$, $V(T)_3 = \{e\}$, $V(T)_4 = \{f\}$, $V(T)_B = \{e, f\}$. Moreover, for example, $\delta(g, T) = 2$, $\Delta(\{a, b, f\}, T) = 6$, $\Delta(V(T)_B, T) = \Delta(\{e, f\}, T) = 7$.

2.2. Definitions

We can now formally define the three problems studied in this paper.

Minimum Branch Vertices Problem (MBV).
Find a spanning tree T of G such that the number of branch vertices is minimized, that is, such that the set $V(T)_B$ has the minimum cardinality.
Minimum Degree Sum Problem (MDS).
Find a spanning tree *T of G* such that the degree sum of the branch vertices is minimized, that is, such that $\Delta(V(T)_B, T)$ is minimized.
Minimum Leaves Problem (ML).
Find a spanning tree *T of G* such that the number of degree-1 vertices is minimized, that is, such that the set $V(T)_1$ has the minimum cardinality.