European Journal of Operational Research 232 (2014) 512–521

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

Multi-period efficiency and Malmquist productivity index in two-stage production systems

Chiang Kao [a,*], Shiuh-Nan Hwang [b]

[a] Department of Industrial and Information Management, National Cheng Kung University, Tainan, Taiwan, ROC
[b] Department of Business Administration, Min Chuan University, Taipei, Taiwan, ROC

ARTICLE INFO

Article history:
Received 22 March 2013
Accepted 22 July 2013
Available online 30 July 2013

Keywords:
Data envelopment analysis
Malmquist productivity index
Efficiency
Two-stage system
Parallel system

ABSTRACT

Conventional two-stage data envelopment analysis (DEA) models measure the overall performance of a production system composed of two stages (processes) in a specified period of time, where variations in different periods are ignored. This paper takes the operations of individual periods into account to develop a multi-period two-stage DEA model, which is able to measure the overall and period efficiencies at the same time, with the former expressed as a weighted average of the latter. Since the efficiency of a two-stage system in a period is the product of the two process efficiencies, the overall efficiency of a decision making unit (DMU) in the specified period of time can be decomposed into the process efficiency of each period. Based on this decomposition, the sources of inefficiency in a DMU can be identified. The efficiencies measured from the model can also be used to calculate a common-weight global Malmquist productivity index (MPI) between two periods, in that the overall MPI is the product of the two process MPIs. The non-life insurance industry in Taiwan is used to verify the proposed model, and to explain why some companies performed unsatisfactorily in the specified period of time.

1. Introduction

Two-stage production systems are often encountered in the real world where the system has two stages (processes) connected in series, in which the inputs are supplied from outside to the first process, which then produces some intermediate products for the second process to produce the final outputs. Many applications of this type of system have been reported, for example, commercial banks in Bangladesh (Akther et al., 2013), the sustainable design performance of automobile manufacturers in the US (Chen et al., 2012), the development of China's coastal areas (Chiu et al., 2011), airline companies in the US (Lu et al., 2012), and mutual fund families in the US (Premachandra et al., 2012). Cook et al. (2010) and Kao and Hwang (2010) present reviews of the related works.

Fig. 1 shows the structure of a two-stage production system, where X_i, $i = 1, \ldots, m$, Z_f, $f = 1, \ldots, g$, and Y_r, $r = 1, \ldots, s$, are the inputs, intermediate products, and outputs, respectively. If the internal processes are ignored, that is, one merely looks at the inputs used by the system, X_i, and the outputs produced by the system, Y_r, then the data envelopment analysis (DEA) technique proposed by Charnes et al. (1978) can be used to measure the relative efficiency of a set of n units, referred to as decision making units (DMUs), with the same type of production. The model is:

$$\begin{aligned} E_k^{CCR} = \max \quad & \sum_{r=1}^{s} u_r Y_{rk} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i X_{ik} = 1 \\ & \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0, \quad j = 1, \ldots, n \\ & u_r, v_i \geqslant \varepsilon, \quad r = 1, \ldots, s, \; i = 1, \ldots, m, \end{aligned} \tag{1}$$

where u_r and v_i are virtual multipliers and ε is a small non-Archimedean number used to avoid ignoring any factor in calculating efficiency (Charnes and Cooper, 1984). Since the efficiency is measured under the most favorable conditions of the DMU being evaluated, the results are persuasive and acceptable by all DMUs.

Without taking the operations of the internal processes into account, an unreasonable result may occur in which the system is efficient while the two processes are not. For this reason, several models that take the operations of the processes into consideration when measuring the system efficiency have been developed, with the network model of Färe and Grosskopf (2000) being the most widely applied. Kao (2009a) classified such models into independent, connected, and relational ones. Recent studies focus on relational models, because they are able to measure the system and

* Corresponding author.
E-mail addresses: ckao@mail.ncku.edu.tw (C. Kao), snhwang@mail.mcu.edu.tw (S.-N. Hwang).

0377-2217/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ejor.2013.07.030

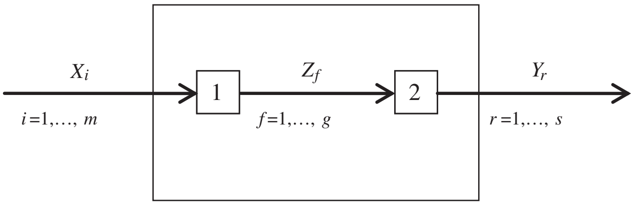

Fig. 1. Two-stage production system.

process efficiencies at the same time, and there is a mathematical relationship between them (Chen et al., 2009; Kao and Hwang, 2008; Tone and Tsutsui, 2009). No matter which model is used, the objective is usually to measure the efficiency of a DMU in a specified period of time.

When the period of time is composed of clearly defined units, such as years, then the data for the inputs, intermediate products, and outputs over the specified period of time can be aggregated in the analysis. For example, Kao and Hwang (2008) used two-year totals to evaluate the efficiency of non-life insurance companies in Taiwan. More often, the average data is used, and one example is Portela et al. (2012), which uses the average results of basic education exams in 2005 and 2006 to measure the performance of Portuguese secondary schools. Since DEA has a unit-invariant property (Lovell and Pastor, 1995), the efficiencies measured from these two types of data, total and average, are the same. If the aggregate data is used, then the resulting efficiency is an overall measure of the performance of the specified period of time. Similar to the case that ignoring the operations of individual processes will result in misleading results, ignoring the operations of individual periods may also produce unreasonable ones. For example, the results may suggest that a DMU is efficient in an overall sense, although every period is inefficient. While one can use the existing two-stage models to calculate the efficiency of each period independently, and then aggregate them to represent the overall performance, there is still a problem with regard how to aggregate the efficiency of individual periods. This paper thus proposes a multi-period model which is able to measure the overall and period efficiencies at the same time, and derive a mathematical relationship between them. The resulting relationship indicates that a DMU in the specified period of time is efficient only if it is efficient in all individual periods.

The proposed model is also able to calculate a common-weight global Malmquist productivity index (MPI) for measuring performance changes between two periods. Since this MPI uses the same frontier facet to calculate efficiencies, the results are more comparable among different DMUs as compared to those calculated from the conventional MPI.

This paper is organized as follows. In the next section, a model for measuring the overall and period efficiencies at the same time is developed. This model may produce multiple solutions, and an approach for determining a solution is described in Section 3. After that, a common-weight global MPI is proposed in Section 4 to measure changes in performance between two periods. An example of the non-life insurance industry in Taiwan is then used to illustrate this approach in Section 5. Finally, in Section 6, some conclusions are drawn from the discussion.

2. Efficiency measurement

Suppose the time span for measuring the efficiency of a set of n DMUs, that have a two-stage structure, covers q periods. Let $X_{ij}^{(p)}$, $Z_{fj}^{(p)}$, and $Y_{rj}^{(p)}$ denote the inputs, intermediate products, and outputs of DMU j in period p, respectively, with totals of $X_{ij}=\sum_{p=1}^{q}X_{ij}^{(p)}$, $Z_{fj}=\sum_{p=1}^{q}Z_{fj}^{(p)}$, and $Y_{rj}=\sum_{p=1}^{q}Y_{rj}^{(p)}$. Fig. 2 is a graphical representation of such a system, and is referred to as a multi-period two-stage production system.

The relational model in Kao and Hwang (2008) for measuring the system (overall) and process efficiencies of the kth DMU for two-stage systems is to take the operations of the two processes into consideration in addition to that of the system defined in the conventional DEA Model (1), where the operation is described by the constraint of requiring the aggregate output to be less than or equal to the aggregate input:

$$\begin{aligned}
\widehat{E}_k^S = \max\ & \sum_{r=1}^{s} u_r Y_{rk} \\
\text{s.t.}\ & \sum_{i=1}^{m} v_i X_{ik} = 1 \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0, \quad j=1,\ldots,n \\
& \sum_{f=1}^{g} w_f Z_{fj} - \sum_{i=1}^{m} v_i X_{ij} \leqslant 0, \quad j=1,\ldots,n \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{f=1}^{g} w_f Z_{fj} \leqslant 0, \quad j=1,\ldots,n \\
& u_r, v_i, w_f \geqslant \varepsilon, \quad r=1,\ldots,s, \quad i=1,\ldots,m, \quad f=1,\ldots,g
\end{aligned} \tag{2}$$

Since the sum of the third and fourth constraints is equal to the second constraint for each DMU, the second set of constraints are redundant, and can be deleted. After an optimal solution $\left(u_r^*, v_i^*, w_f^*\right)$ is obtained, the system efficiency, $\widehat{E}_k^S$, and two process efficiencies, $\widehat{E}_k^I$ and $\widehat{E}_k^{II}$, are calculated as:

$$\begin{aligned}
\widehat{E}_k^S &= \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
\widehat{E}_k^I &= \sum_{f=1}^{g} w_f^* Z_{fk} \Big/ \sum_{i=1}^{m} v_i^* X_{ik} \\
\widehat{E}_k^{II} &= \sum_{r=1}^{s} u_r^* Y_{rk} \Big/ \sum_{f=1}^{g} w_f^* Z_{fk}
\end{aligned} \tag{3}$$

Clearly, the system efficiency is the product of the two process efficiencies, $\widehat{E}_k^S = \widehat{E}_k^I \times \widehat{E}_k^{II}$.

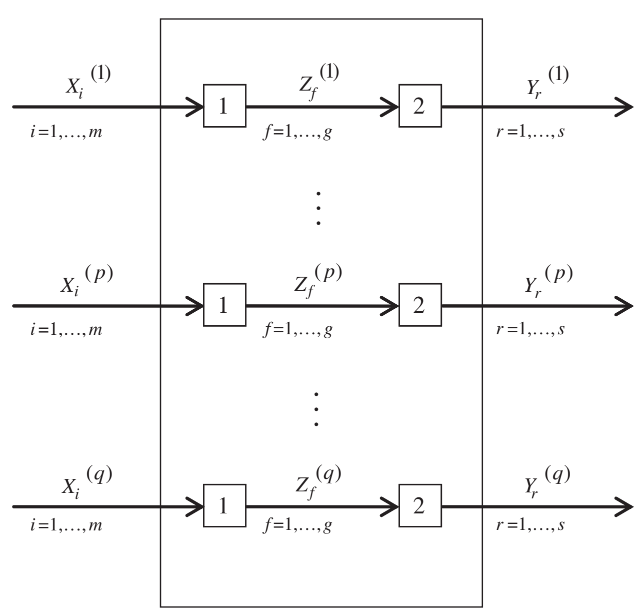

Fig. 2. Multi-period two-stage production system.