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Decision Support

Approximate decomposition methods for the analysis of multicommodity flow routing in generalized queuing networks

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A B S T R A C T

In this study we deal with network routing decisions and approximate performance evaluation approaches for generalized open queuing networks (OQN), in which commodities enter the network, receive service at one or more arcs and then leave the network. Exact performance evaluation has been applied for the analysis of Jackson OQN, where the arrival and service processes of the commodities are assumed to be Poisson. However, the Poisson processes' hypotheses are not a plausible or acceptable assumption for the analysis of generalized OQN, as their arrival and service processes can be much less variable than Poisson processes, resulting in overestimated system performance measures and inappropriate flow routing solutions. In this paper we merge network routing algorithms and network decomposition methods to solve multicommodity flow problems in generalized OQN. Our focus is on steady-state performance measures as average delays and waiting times in queue. The main contributions are twofold: (i) to highlight that solving the corresponding multicommodity flow problem by representing the generalized OQN as a Jackson OQN may be a poor approximation and may lead to inaccurate estimates of the system performance measures, and (ii) to present a multicommodity flow algorithm based on a routing step and on an approximate decomposition step, which leads to much more accurate solutions. Computational results are presented in order to show the effectiveness of the proposed approach.

1. Introduction

In this study we deal with routing multicommodity flows in open queuing networks (OQN) in which commodities enter the network, receive service at one or more arcs and leave the network. There are different studies in the literature representing manufacturing, distribution, communication and other systems as OQN, e.g., Buzacott and Shanthikumar (1993), Gershwin (1993), Nakano and Ohno (1999), Bitran and Morabito (1999), Kerbache and Smith (2000), Ouorou, Mahey, and Vial (2000), Warsing, Souza, and Greis (2001), Kerbache, Norbis, and Gonzalez (2004), Kerbache and Smith (2004), Azaron, Katagiri, Kato, and Sakawa (2006), Ouorou (2007), Wille, Mellia, Leonardi, and Marsan (2009), Smith (2010), Smith, Cruz, and van Woensel (2010), Koo, Koh, and Lee (2011), Liu, Yang, Wan, and Fowler (2011), Bedell and Smith (2012), Ishfaq and Sox (2011), Wu and McGinnis (2012, 2013) and the references therein.

Kerbache and Smith (2000) addressed the problem of optimal routing in generalized queueing networks arising in the context of material handling. The physical material handling system is broken down into a set of layout topologies, and the problem is multi-objective since the product throughput is maximized while minimizing the average sojourn time and holding costs. The authors proposed a methodology combining optimization and analytical queueing network models to select the set of routes that would minimize the routing costs as well as the average sojourn times for each product class. Related research in routing was developed for flexible manufacturing systems (Yao & Buzacott, 1985); for production and manufacturing settings (Daskalaki & Smith, 1989); and for emergency evacuation (Bakuli & Smith, 1996; Stepanov & Smith, 2009). Kerbache and Smith (2004) examined a hierarchy of linked supply chain modelling systems to support operational, tactical and strategic decision making. The authors developed analytical queueing networks coupled with nonlinear optimization methods to design supply chain topologies and evaluate their various performance measures.

The motivation from the modelling point of view for this study comes from data communication networks. In this context, an important problem in packet-switched computer networks consists in determining routes on which packets have to be transmitted in order to optimize a performance measure on quality of service when operating the network; see, e.g., Kleinrock (1964),

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Fratta, Gerla, and Kleinrock (1973), Gerla and Kleinrock (1977), Bertsekas and Gafni (1983), Bertsekas and Gallager (1987), Gavish and Neuman (1989), Chifflet, Mahey, and Reyner (1994), Mahey, Ouorou, LeBlanc, and Chifflet (1998), Ouorou et al. (2000), Ouorou (2007), de Souza, Mahey, and Gendron (2008), Wille et al. (2009).

The network is represented by a directed graph, and it is given a set of ordered origin–destination node pairs. An origin node "creates" a traffic demand (for example, compressed video images) to be sent through the network and "consumed" at the destination node. Thus, between each pair there is a traffic, measured in data units/time units, entering and exiting the network at the origin and destination, respectively. The traffic can be decomposed into packets which can take any number of routes to flow from its origin node to its destination node. A route is an elementary path on the graph representing the network. Denoting by i_1 the origin node, and by i_b the destination node, an elementary path from i_1 to i_b is a sequence $i_1 - (i_1, i_2) - i_2 - (i_2, i_3) - \ldots - (i_{b-1}, i_b) - i_b$ of b nodes and $b - 1$ arcs on the graph starting at i_1 and ending at i_b without any repetition of nodes or arcs. It may happens $b = 2$, and the path consists of two nodes and a single arc; otherwise, a node i_a, $a = 2, \ldots, b - 1$ is said to be an intermediate node. To be transmitted from i_1 to i_b, a packet may wait in queue at node i_1 to go through arc (i_1, i_2), then may wait in queue at node i_2 to go through arc (i_2, i_3), and so on throughout the route until reach node i_b.

We take into account modelling assumptions that have been considered in the literature on optimization of packet-switched networks cited above. It has been considered that between each pair of nodes there is only one arc in the network, and that it transmits one packet at a time. An arc corresponds to a machine in a more usual queuing context (a detailed correspondence between network flows and queuing networks terminologies is provided in Section 2). Thus, the model has a single server, i.e., an arc, at each station that treats one recipe at a time. Because every route on which packets go through are elementary paths, the network is no reentrant. The problem is modelled as an open queuing network since packet-switched networks operate on a point-to-point basis, i.e., a traffic entering the network is sent to an specific destination where it exits the network (the transmission is finished after reaching the destination node). For such reason, even though the queuing literature have a number of studies representing systems as closed queuing networks, in which for each external departure there is an external arrival so that the number of customers in the network remains constant, we keep the focus of the present study upon an open queuing model. We also remark that, to the best of our knowledge, multicommodity flow problems in packet-switched networks have been usually studied in the literature considering unlimited buffer at the nodes of the network.

Average delay has been extensively used as a major system performance measure in data communication networks. The literature on optimization of packet-switched networks has been concentrated so far on Jackson networks (Jackson, 1957, 1963), where the arrival and service processes are assumed to be Poisson. And exact performance evaluation has been applied, as each arc of the network can be analysed individually as a stochastically independent M/M/1 queuing system. Consequently, the resulting multicommodity flow problems arising from optimization studies on packet-switched networks have nonlinear convex costs separable on each arc, which allows the use of efficient methods (Ouorou et al., 2000).

In many systems, however, the Poisson processes' hypotheses may not be a plausible or acceptable assumption. We believe that multicommodity flow problems subjected to less variable arrival and/or service processes than the Poisson process are worth studying. Certainly, flow problems in an environment where the variabilities of the interarrival and service times are higher than Poisson processes (i.e., with squared coefficients of variation higher than 1) merit attention, but our focus on this paper are on less variable processes than Poisson and, as far as we know, this is one of the first studies of this kind in the multicommodity flow routing literature. Manufacturing, distribution, communication and other systems have in general arrival and service processes less variable than the Poisson process and the representation of these systems as Jackson networks generally results in overestimated performance measures, e.g., Bitran and Tirupati (1988); Jiang and Giachetti (2008); Silva and Morabito (2009).

In the absence of exact methods for the analysis of generalized OQN where the interarrival and service times have general probability distributions, different authors have developed approximate methods for performance measures evaluation of these networks. Two relevant approaches for generalized OQN are based on the stochastic independence (Whitt, 1983a, 1983b, 1984) and heavy traffic (Harrison & Nguyen, 1990) assumptions. Both of them rely on the independence assumption either directly or indirectly as the heavy traffic approach relies on the functional central limit theorem. In particular, approximate decomposition methods have been largely used (Bitran & Morabito, 1996; Buzacott & Shanthikumar, 1992; Disney & Konig, 1985; Whitt, 1984), which is also based on the stochastic independence assumption. For instance, Bitran and Morabito (1996) reviewed developments of such methods combining them with mathematical programming techniques in optimization problems for the design and planning of discrete manufacturing systems (job shop systems in particular). The studied manufacturing systems are analysed as single or multiple product class generalized OQN with deterministic and probabilistic routing. In general the decisions are not in optimizing the flow routing, but rather: (i) in minimizing total capacity investments at the network workstations subject to attaining a targeted network performance measure (e.g., a targeted work-in-process level or product leadtime), and (ii) in optimizing system performance levels subject to budget constraints (e.g., for the resource costs and capacity investments at the workstations) (Bitran & Morabito, 1996, 1999). Other studies appear in Wu and McGinnis (2012, 2013) based on the concept of the intrinsic ratio, which gives reliable waiting time approximations for OQN.

In this paper we cope with network routing decisions and approximate performance evaluation approaches for generalized OQN. Our aim here is to merge routing algorithms and approximate decomposition methods to solve multicommodity flow problems in generalized OQN. Our focus is on steady-state performance measures as average delays and waiting times in queue. The main contributions of this study are twofold: (i) to highlight that solving a multicommodity flow problem by representing the network as a Jackson OQN, as it is common in literature, may be a poor approximation and may lead to inaccurate estimates of the system performance measures if the network is in fact a generalized OQN, and (ii) to propose a multicommodity flow algorithm based on a routing step and on a performance evaluation by approximate decomposition step, which leads to much more accurate performance measure estimates for generalized OQN. We are not aware of other studies in the literature exploring this line of research.

The paper is structured as follows. In Section 2 we briefly define the network flow routing problem, present an arc-path multicommodity flow model and discuss Jackson and generalized OQN. We also illustrate the correspondence between the terminologies of queuing networks and multicommodity network flows. In Section 3 we present a pseudo-code of the proposed approach based on a cycle cancelling algorithm for the convex multicommodity flow problem and on three different approximate decomposition methods for performance evaluation of generalized OQN. In Section 4 we analyse the numerical results of computational experiments evaluating the benefits of the proposed approach with respect to the traditional approach based on the assumption of a Jackson