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Innovative Applications of O.R.

A computational analysis of multidimensional piecewise-linear models with applications to oil production optimization ☆

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A B S T R A C T

Production optimization of gas-lifted oil wells under facility, routing and pressure constraints is a challenging problem, which has attracted the interest of operations engineers aiming to drive economic gains and scientists for its inherent complexity. The hardness of this problem rests on the non-linear characteristics of the multidimensional well-production and pressure-drop functions, as well as the discrete routing decisions. To this end, this work develops several formulations in Mixed-Integer Linear Programming (MILP) using multidimensional piecewise-linear models to approximate the non-linear functions with domains spliced in hypercubes and simplexes. Computational and simulation analyses were performed considering a synthetic but realistic oil field modeled with a multiphase-flow simulator. The purpose of the analyses was to assess the relative performance of the MILP formulations and their impact on the simulated oil production.

1. Introduction

Although technological advances have increased the diversity of energy resource alternatives, fossil energy remains a primal source of energy in the modern world. In response to the increasing demand for petroleum and highly competitive markets, the petroleum industry has been investing in innovation aiming to optimize production processes and cut operational costs.

On the daily operation of oil fields several decisions are made to determine the production plan: valve configurations, well-manifold routings, and choke openings, among others. Further, the engineers have to deal with uncertainties and complex problems that arise from unexpected contingencies, such as compressor failure and choke stalling caused by equipment wear and abrasive fluid production. In existing oil fields, operational plans are reached promptly often based on a sensitivity analysis that uses simulation tools and heuristics. Even though these methods can obtain good operational plans, they do not necessarily ensure a mode of operation that maximizes the daily production.

In principle, optimal production plans could be achieved by formulating the production optimization problem as a Mixed-Integer Nonlinear Program (MINLP) accounting for complex nonlinear phenomena like well production and pressure drop. Since these relations are not known explicitly, they can be approximated using nonlinear functions to cast the problem as an MINLP or using piecewise-linear models to yield a Mixed-Integer Linear Programming (MILP) problem.

Although several works have applied mathematical programming methods to obtain optimal production plans (Buitrago, Rodríguez, & Espin, 1996; Alarcón, Torres, & Gómez, 2002; Camponogara & de Conto, 2009; Misener, Gounaris, & Floudas, 2009; Codas & Camponogara, 2012), only a limited number of works considered pressure drop in pipelines which cannot be neglected when the operating conditions vary due to routing operations and equipment failure, for instance.

Beggs and Brill (1973) were among the first to study the representation of pressure in oil production systems, who proposed correlations between the pressure drop and flow in pipelines. Litvak and Darlow (1995) developed analytic and piecewise-linear models for representing pressure equations. The first represents more precisely the physical phenomenon and is routinely used in software for simulating flows in oil production systems. The second technique approximates pressure drops as a function of the outlet pressure and the oil, gas, and water flows.

More recent works in oil production optimization considering complex operations, such as routing and pressure drop, can be roughly divided in MINLP and MILP methods. Kosmidis, Perkins, and Pistikopoulos (2004, 2005) present an MINLP optimization formulation of a production network with naturally producing and gas-lifted wells. The MINLP problem is solved by a sequence of MILP problems following a sequential linear programming strategy. On the other hand, Gunnerud and Foss (2010) present an MILP

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formulation for oil fields structured in clusters of independent wells, manifolds, and pipelines while the separation facilities are centralized in a platform. Piecewise linearization techniques based on special ordered sets of type 2 (SOS2) constraints are used to approximate nonlinear functions.

Because the complex behavior of well production and pipeline pressure drop are not explicitly known, the synthesis of an MINLP program will entail fitting sample data from a simulator to nonlinear models, a task that itself is challenging and dependent on the space of models considered. On the other hand, piecewise-linear models have the advantage of not needing the synthesis of such relationships, being defined directly from the sample points, a property that motivates the work herein.

Although some works have represented pressure drops with piecewise-linear models, a comprehensive analysis of the existing models for piecewise linearization in oil production optimization is lacking. To this end, this work presents a computational and simulation analysis of piecewise-linear models for hypercube- and simplex-based approximations of the function domains, considering an oil field modeled with a standard multiphase-flow simulator.

The work is organized as follows. Section 2 defines the problem of optimizing gas-lifted oil fields as an MINLP problem. Section 3 offers a brief review of piecewise-linear functions and several MILP approximation formulations of the gas-lift optimization problem. Section 4 presents a synthetic but realistic model of an off-shore oil field, along with a computational analysis of the developed MILP formulations and a simulation analysis considering both hypercube- and simplex-based approximations. Finally, Section 5 presents some conclusions and suggests directions for future research.

2. Problem definition

The problem of optimizing the daily production of a gas-lifted oil field taking into account constraints on lift-gas availability, separation capacity, well-manifold routing, and pressure constraints can be formulated as a mixed-integer non-linear program:

$$P: \max f = \sum_{m \in \mathcal{M}} g(\mathbf{q}^m) - \sum_{n \in \mathcal{N}} c(q_i^n) \tag{1a}$$

$$\text{s.t.}: \sum_{n \in \mathcal{N}} q_i^n \leqslant q_i^{\max} \tag{1b}$$

For all $n \in \mathcal{N}$:

$$q_i^{n,\min} y_n \leqslant q_i^n \leqslant q_i^{n,\max} y_n \tag{1c}$$

$$\sum_{m \in \mathcal{M}_n} z_{n,m} = y_n \tag{1d}$$

$$\mathbf{q}^{n,m} = \mathbf{q}^{n,m}(p^m, q_i^n) z_{n,m}, \ \forall m \in \mathcal{M}_n \tag{1e}$$

$$z_{n,m} \mathbf{q}^{n,\mathrm{L}} \leqslant \mathbf{q}^{n,m} \leqslant z_{n,m} \mathbf{q}^{n,\mathrm{U}}, \ \forall m \in \mathcal{M}_n \tag{1f}$$

$$\mathbf{q}^m = \sum_{n \in \mathcal{N}_m} \mathbf{q}^{n,m} \leqslant \mathbf{q}^{m,\mathrm{S}}, \ \forall m \in \mathcal{M} \tag{1g}$$

$$p^m = p^{m,\mathrm{S}} + \Delta p^m(\mathbf{q}^m), \ \forall m \in \mathcal{M} \tag{1h}$$

$$p^{m,\min} \leqslant p^m \leqslant p^{m,\max} \tag{1i}$$

$$0 \leqslant y_n \leqslant 1, \ \forall n \in \mathcal{N} \tag{1j}$$

$$z_{n,m} \in \{0,1\}, \ \forall n \in \mathcal{N}, \ \forall m \in \mathcal{M}_n \tag{1k}$$

with **decision variables** for optimization being:

- q_i^n is the lift-gas rate injected into well n;
- y_n takes on value 1 when well n is producing and 0 otherwise. Dispite being a continuous variable it can only assume values 1 or 0 as a consequence of Eq. (1d);
- $z_{n,m}$ assumes value 1 if the production of well n is directed to manifold m, and 0 otherwise;
- $q_h^{n,m}$ is the flow of phase $h \in \mathcal{H}$ directed from well n to manifold m, and $\mathbf{q}^{n,m} = (q_h^{n,m} : h \in \mathcal{H})$ is the vector of all phase flows;
- $\mathbf{q}^m = \sum_{n \in \mathcal{N}_m} \mathbf{q}^{n,m}$ is the total flow received by manifold m;
- p^m is the pressure of manifold m;

with **parameters**:

- N is the number of oil wells, $\mathcal{N} = \{1, \ldots, N\}$, and $\mathcal{N}_m \subseteq \mathcal{N}$ is the subset of wells whose production can be directed to manifold m;
- M is the number of manifolds, $\mathcal{M} = \{1, \ldots, M\}$ and $\mathcal{M}_n \subseteq \mathcal{M}$ is the subset of manifolds that can receive production from well n. The production of each manifold is processed by a dedicated separator;
- $\mathcal{H} = \{\mathrm{o}, \mathrm{g}, \mathrm{w}\}$ has the phase flows, namely oil (o), gas (g), and water (w);
- $q_i^{\max}$ models the lift-gas that can be delivered by the compression station;
- $q_i^{n,\min}$ and $q_i^{n,\max}$ are bounds on the lift-gas injection for well n typically used to ensure production stability, avoid slugging, and follow a recovery policy for the reservoir;
- $p^{m,\mathrm{S}}$ is the operational pressure of the separator connected to manifold m;
- $\mathbf{q}^{n,\mathrm{L}}$ and $\mathbf{q}^{n,\mathrm{U}}$ are vectors with lower and upper bounds on the production of well n;
- $\mathbf{q}^{m,\mathrm{S}}$ is the processing capacity of the separator of manifold m;
- $p^{m,\min}$ ($p^{m,\max}$) is the minimum (maximum) operational pressure for manifold m;

and with **functions**:

- f is a function composed by a function g that represents the economic benefit from oil production and a function c which represents the lift-gas injection cost, however any other continuous function can be considered such as the total oil production;
- $q_h^{n,m}(p^m, q_i^n)$ is the flow of phase h sent by well n to manifold m given as a function of the manifold pressure and lift-gas injection, and $\mathbf{q}^{n,m}(p^m, q_i^n) = (q_h^{n,m}(p^m, q_i^n) : h \in \mathcal{H})$ is the vector of all phase flows;
- $\Delta p^m(\mathbf{q}^m)$ represents the pressure drop in the pipeline connecting manifold m to its adjoint separator.

Among the decision variables for optimization, the ones that are actually controlled in the oil field are the well activation decisions y_n, the routing decisions $z_{n,m}$, and the lift-gas rates q_i^n.

The nonlinearity in the production optimization problem arises from the nonlinear nature of the well-production function $q_h^{n,m}$ and the pressure drop Δp^m in pipelines, which later will be approximated with piecewise-linear models. This problem extends the work of Codas and Camponogara (2012) by explicitly modeling pressure drops and considering pressure constraints.

The gas compressing capacity and the bound on lift-gas injection for the wells are defined by constraints (1b) and (1c) respectively.

The well-manifold routing constraints (1d) and (1e) ensure that the production of each well will be sent to precisely one manifold when the well is producing, *i.e.* $y_n = 1$. The production of well n sent to manifold m is zero if not routed to this manifold, *i.e.* $z_{n,m} = 0$, however the production becomes bounded by $\mathbf{q}^{n,\mathrm{L}}$ and $\mathbf{q}^{n,\mathrm{U}}$ if well n is routed to manifold m.

The operational limits for well production, the mass balance equations, and separation capacity are established by constraints (1f) and (1g), while the pressure balance between manifolds and separators are given by constraints (1h) and (1i).

Fig. 1 illustrates the structure of the oil production system and the semantics of the decision variables. The available lift-gas rate