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Continuous Optimization

A new local search for continuous location problems ☆

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ABSTRACT

This paper presents a new local search approach for solving continuous location problems. The main idea is to exploit the relation between the continuous model and its discrete counterpart. A local search is first conducted in the continuous space until a local optimum is reached. It then switches to a discrete space that represents a discretisation of the continuous model to find an improved solution from there. The process continues switching between the two problem formulations until no further improvement can be found in either. Thus, we may view the procedure as a new adaption of *formulation space search*. The local search is applied to the multi-source Weber problem where encouraging results are obtained. This local search is also embedded within Variable Neighbourhood Search producing excellent results.

1. Introduction

Location models generally require finding the location of a given number, say p, of new facility sites in order to serve in some optimal way (e.g., minimum cost) a given set of existing facilities, also known as customers or demand (or fixed) points. If the model is formulated in continuous space, a distance function is required to calculate the distance between pairs of points. Since the new facilities may be located anywhere in the continuous space or regions thereof, these models are referred to as site generating models (e.g., see Love, Morris, & Wesolowsky, 1988). The distance functions most commonly used are the Euclidean norm and the rectangular (or Manhattan) norm; however, more sophisticated models of distance are available when more accurate estimates of actual travel distances are desired (e.g., Brimberg & Walker, 2010).

The same location problem may be formulated in discrete space by restricting the potential new facility sites to a specified finite set of points in the continuous space. If these sites are chosen well, and a good algorithm or heuristic is available to solve the discrete formulation, we may anticipate a ''good'' solution to the original problem. For example, if we restrict the candidate facility sites to the given set of fixed points, the classical multi-source Weber problem, also known as the continuous location-allocation problem, converts to the classical (discrete) p-median problem. We may then try to obtain a good solution to the discrete model, and use it as a starting point for the continuous model.

Exploiting the relation between the p-median model and the continuous location-allocation model has been suggested as early as in the original work of Cooper (1963, 1964). Hansen, Mladenović, and Taillard (1998) tested a heuristic that first solves the p-median problem exactly using a primal–dual algorithm by Erlenkotter (1978), and then completes one iteration of ''continuous-space adjustment'' by solving the p continuous single facility problems identified in the first phase. Brimberg, Hansen, Mladenović, and Taillard (2000) examined this heuristic among others, and concluded that computation time became a limiting factor on larger problem instances. Gamal and Salhi (2003) used a similar approach where in the first phase, an effective heuristic is applied instead of an exact solution approach to solve the p-median problem.

In certain cases it may be shown that the continuous problem has a finite dominating set. For example, if the rectangular norm (l_1-norm) is used as the distance function, it is well known that an optimal solution of the continuous location-allocation problem exists where each of the facilities is located at a vertex of a grid formed by drawing horizontal and vertical lines (the fundamental directions of the l_1-norm) through each of the demand points. Since an optimal solution in the plane must also exist with all facilities inside the convex hull of the demand points (e.g., see Hansen, Perreur, & Thisse, 1980), or the smaller rectangular hull for the l_1-norm (Love et al., 1988), a discrete formulation of the continuous location-allocation problem with a fewer number of nodes is also

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possible that guarantees an optimal solution of the original problem. This idea can be extended to the class of polyhedral (or block) norms (e.g., Ward & Wendell, 1980,Ward, Wendell, & Richard (1985)), although the grid will be more complicated in general due to a higher number of fundamental directions attributed to the norm. This discretisation of the continuous space does not extend to round metrics such as the Euclidean norm.

In practice the discrete formulation, whether or not it contains an optimal solution of the original continuous location problem, may become rather large to be tackled optimally. Aras, Altinel, and Orbay (2007) propose a discrete approximation to solve the capacitated multi-source Weber problem (CMSWP) with Euclidean, squared Euclidean and l_p distances with $1 < p < 2$. The authors discretise the solution space while increasing the number of potential sites by using the rectangular grid points that are within the convex hull of the customers. Two MILP formulations are proposed using this new set of potential sites, including some attempts in choosing a subset. Heuristic approaches such as a Lagrangean relaxation-based method, the $p-$median heuristic of Hansen et al. (1998) and the cellular Heuristic of Gamal and Salhi (2001) are also investigated. Aras, Orbay, and Altinel (2008) adapt the previous approaches to the case of rectilinear distances whereas Durmaz, Aras, and Altinel (2009) extend this discretisation approach to cater for uncertainty due to changes in the customer set. Very recently, Akyüz, Altinel, and Öncan (2013) studied the CMSWP using two branch-and-bound techniques where one is related to the discretisation of the location space. Heuristics based on solving the discrete approximation of the CMSWP by Lagrangean Relaxation are proposed by Boyaci, Altinel, and Aras (2013).

For an overview of the continuous location-allocation problem, the interested reader is referred to the survey paper by Brimberg, Hansen, Mladenovic, and Salhi (2008) and the references therein, while for the discrete p-median model and solution approaches the review by Mladenović, Brimberg, Hansen, and Moreno-Perez (2007) can be useful.

The relation between discrete and continuous formulations may be extended to many other location models. For example, the less-studied continuous p-centre problem becomes the better-known discrete p-centre problem when candidate facility sites are once again restricted to the set of fixed points. The classical (discrete) simple plant location problem has more recently been modelled in continuous space by Brimberg and Salhi (2005), and in a related paper by Brimberg, Mladenovic, and Salhi (2004). Indeed, the idea of exploring the relation between discrete and continuous location problems presents in our view a rich new area of research.

In this paper we present a new local search for solving continuous location problems that is based on reformulations of the problem in continuous and discrete space. The basic idea is to find a local optimum in continuous space using any convenient local search algorithm. The search space is then modified by reformulating the problem in discrete space. Here we introduce the idea of augmenting a specified set of fixed points (the current set) with the local optima obtained in the continuous phase. Thus, we solve exactly or heuristically a discrete problem where the nodes of the network now include the new facility sites obtained in the previous step. We switch back to continuous space using the discrete solution as the starting point. The procedure alternates between continuous and discrete spaces, always adding newly acquired facility sites to the current set in the discrete formulation, until no further improvement is found.

The local search outlined above incorporates elements of a metaheuristic known as formulation space search (FSS). The basic idea here as presented in Mladenović, Plastria, and Urosevic (2005) is to use different formulations of a combinatorial or global optimization problem in an iterative fashion, where in each formulation suitable local searches are used. For example, the authors applied two formulations in different coordinate systems of the circle packing problem with excellent results.

In formulation space search, the different formulations are all equivalent to each other. However, in our case, the discrete model is an approximation of the continuous model, thus presenting a fundamental departure from FSS. We may also argue, meanwhile, that the discrete formulation is equivalent to the continuous formulation in an asymptotic sense, as more facility sites generated in the continuous phase are added to the network.

The paper is organized as follows. In the next section we provide a basic framework for the proposed local search. Section 3 illustrates the local search on the multi-source Weber problem (MWP) using a well-known 50-customer problem from the literature. Larger problem instances of MWP are examined later in this section. Section 4 develops a variable neighbourhood search (VNS) heuristic for solving MWP that employs the proposed local search in its local search step. The same data sets are also tested here and superior computational results are obtained. The last section summarizes our conclusions and highlights some suggestions for further research.

2. The local search

We consider an unconstrained location problem of the general form

$$\min\ f(X_1, X_2, \ldots, X_p). \tag{GLP}$$

where $X_i \in \mathbb{R}^N$ gives the unknown location of new facility i, $i = 1, \ldots, p$, and the objective function $f(.)$ represents some performance measure, such as total cost. Typically the location problem occurs in the plane, so that $N = 2$, and X_i is given by the Cartesian coordinates (x_i, y_i).

Consider as an illustration the classical multi-source Weber problem, which may be formulated as follows:

$$\min\ f(X_1, X_2, \ldots, X_p) = \sum_{j=1}^{n} w_j \min_{i=1,\ldots,p}\{\|X_i - A_j\|\}. \tag{MWP1}$$

Here A_j denotes the known coordinates of customer j, $w_j > 0$, the known demand at A_j, and $\|X_i - A_j\|$ the Euclidean distance between the pair of points X_i and A_j, $i = 1, \ldots, p$, $j = 1, \ldots, n$. The objective function gives a sum of weighted distances from the demand points to their nearest facilities, and thus, represents a measure of the total cost of the current solution.

As a second illustration, consider the continuous weighted p-centre problem, which may be formulated as follows:

$$\min\ g(X_1, X_2, \ldots, X_p) = \max_{j=1,\ldots,n}\{v_j \min_{i=1,\ldots,p}\{\|X_i - A_j\|\}\}, \tag{MCP}$$

where weight $v_j > 0$ reflects the "importance" of demand point A_j, and the remaining notation is the same as for ($MWP1$). The objective function gives the maximum (weighted) distance between the demand points and their nearest facilities, and thus, represents a measure of the quality of service of the current solution.

Other examples include the use of ordered medians (e.g., Nickel & Puerto, 2005), or the use of negative-valued weights in ($MWP1$) or (MCP) to model obnoxious facilities (e.g., Erkut & Newman, 1989). In the latter case, restrictions on the location of the facilities may be required in order to guarantee that an optimal solution exists. Capacity constraints and flow variables may also be included in the model.

We now describe the basic steps of the proposed local search for problems of type (GLP). To differentiate between the continuous and discrete formulations of the problem, we let (GLP) denote the original continuous formulation and $(GLP)'$ the discrete approximation. Let S denote a finite set of identified potential sites for the new facilities, and X a subset of p of these sites. For example, $S = \{A_1, \ldots, A_n\}$, where typically $n \gg p$, has been recommended in