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Discrete Optimization

Adaptive linear combination of heuristic orderings in constructing examination timetables

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ABSTRACT

In this paper, we investigate adaptive linear combinations of graph coloring heuristics with a heuristic modifier to address the examination timetabling problem. We invoke a normalisation strategy for each parameter in order to generalise the specific problem data. Two graph coloring heuristics were used in this study (largest degree and saturation degree). A score for the difficulty of assigning each examination was obtained from an adaptive linear combination of these two heuristics and examinations in the list were ordered based on this value. The examinations with the score value representing the higher difficulty were chosen for scheduling based on two strategies. We tested for single and multiple heuristics with and without a heuristic modifier with different combinations of weight values for each parameter on the Toronto and ITC2007 benchmark data sets. We observed that the combination of multiple heuristics with a heuristic modifier offers an effective way to obtain good solution quality. Experimental results demonstrate that our approach delivers promising results. We conclude that this adaptive linear combination of heuristics is a highly effective method and simple to implement.

1. Introduction

The examination timetabling problem has been much studied and a wide variety of approaches have been taken across a variety of associated problem descriptions. In general, the task is NP hard (Schindl, 2005). The real world problem is rich and varied, involving significant levels of information (Burke, Elliman, Ford, & Weare, 1996). The manual solution of this problem is typically suboptimal (feasible but not a very good solution) since the exploration of the space for high quality solutions is beyond the scope of ad hoc search. Examination timetabling problems have been well documented in the academic literature with a good coverage of various methods and strategies (Carter, Laporte, & Lee, 1996; Qu, Burke, Mccollum, Merlot, & Lee, 2009).

The examination timetabling problem can be defined as the assignment of a finite set of examinations to a finite set of time-slots whilst, at the same time, satisfying various problem constraints. It involves two types of constraints; hard constraints and soft constraints. The hard constraints are strictly required to be adhered to in any circumstances. Satisfying the hard constraints produces a *feasible* solution. For example, students cannot sit two examinations at the same time. On the other hand, soft constraints do not affect the feasibility of the solution but they need to be satisfied as much as possible for the solution to be of high quality. Of course, soft constraints usually have to be violated to some degree in a real world situation. The extent to which the defined soft constraints are satisfied reflects the quality of the obtained timetable. An example of a soft constraint is that students should have as much time as possible between examinations. An example of real-world application of automated examination timetabling is reported in Kahar and Kendall (2010).

Timetabling approaches have been widely investigated at the interface of Artificial Intelligence and Operations Research over the last few decades or so. The examination timetabling problem can be mapped through an identity relationship onto a graph colouring mathematical formalism. Indeed, this observation underpins some of the earliest and most well known approaches to examination timetabling problems (Carter, 1986). In the graph

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colouring formalism, the vertices represent examinations and the edges connecting vertices represent hard constraint, i.e. conflicts between the examinations. For more details on graph representation in timetabling see Burke, Kingston, and de Werra (2004).

Many other approaches such as meta-heuristics and hybridisations have been successfully investigated. Examples include evolutionary algorithms (Ersoy, Özcan, & Sima Uyar, 2007), tabu search (White, Xie, & Zonjic, 2004), ant algorithms (Naji Azimi, 2005), the Great Deluge approach (Burke, Bykov, Newall, & Petrovic, 2004) and simulated annealing (Thompson & Dowsland, 1996).

Recently, other variants of local search approaches have been widely explored within the context of examination timetabling. These include variable neighbourhood search (Burke, Eckersley, McCollum, Petrovic, & Qu, 2010) and iterated local search (Caramia, Dell'Olmo, & Italiano, 2008). Some recent approaches have been motivated b the goal of developing more general algorithms. Examples include hyper-heuristics (Burke, Mccollum, Meisels, Petrovic, & Qu, 2007), case-based reasoning (Burke, Petrovic, & Qu, 2006), fuzzy approaches (Asmuni, Burke, Garibaldi, McCollum, & Parkes, 2009) and granular information processing (Abdul Rahim, Bargiela, & Qu, 2009). A review of the major approaches in examination timetabling can be found in Carter et al. (1996) and Qu et al. (2009).

The successful assignment of an examination to a time-slot is closely related to the initial ordering strategy in which all examinations are processed. Consequently, examinations are first ordered according to the perceived difficulty of being scheduled in the available time-slots. The examinations are then taken one by one to be assigned to the time-slot. The examination deemed to be the most difficult is scheduled first in the timetable. This is a constructive process. Such processes are often used during the initialisation strategy of a meta-heuristic technique. In the past, there have been various ordering strategies employed in the context of examination timetabling (Carter, 1986; Burke et al., 2004). Commonly used ordering strategies are: saturation degree, largest degree, largest weighted degree, largest enrolment and colour degree.

Since none of the ordering strategies provides a guarantee of successful scheduling, there has been some attention on ordering heuristics within adaptive approaches in the academic literature. In our previous study (Abdul Rahman, Bargiela, Burke, McCollum, & Özcan, 2009), we introduced several strategies to choose examinations and time-slots using ordering heuristics within the framework of squeaky wheel optimisation. This work is an extension of the adaptive heuristic orderings technique proposed by Burke and Newall (2004) where the approach promotes early scheduling of difficult examinations based on a *heuristic modifier*. Another study, Qu, Burke, and McCollum (2009) implemented an adaptive approach to examination timetabling by hybridising the low level graph heuristics based on a learning mechanism and modifying the solutions by high-level heuristic indirectly.

With most of the approaches taken within the overall family of constructive methods, it is often the case that a single heuristic is used during the initial ordering phase. In considering the difficulty of an examination, it is useful to take into account other factors that affect the ordering of examinations. Considering many factors at once represents the real world situation. The difficulty of scheduling an examination can be approximated more reliably if several heuristics lend support to the final ordering of examinations. Consequently, the constructive study by Burke, Pham, Qu, and Yellen (2010) combined graph colouring heuristics with weights within a linear approach as to measure the difficulty of a vertex of weighted graph. The study used the vertex-selection heuristics to represent the difficulty of a vertex which is continually updated throughout the timetabling process. Studies by Johnson (1990) and Asmuni et al. (2009) have also deployed this strategy by considering more than one heuristic at one time and it has been shown to have an effect on the ordering of the examinations. Based on the '*difficulty factor*', Johnson (1990) used graph colouring heuristics, i.e. the combination of largest enrolment and largest degree as an ordering strategy for assigning examinations to time slots. Several variations of relative weight of each criterion were considered in order to produce a number of different feasible timetables. Furthermore, Asmuni et al. (2009) combined two graph colouring heuristics within the framework of fuzzy methodology in order to deal with uncertainty in ordering the examination based on its difficulties. Three graph colouring heuristics were used, i.e. largest degree, largest enrolment and saturation degree with three combinations of two heuristics. The study indicated that the solution quality was superior compared with using only a single heuristic.

Encouraged by these studies, we extend this work by combining heuristics with a heuristic modifier and adapting different weights to each heuristic to analyse its effectiveness. The aim is to obtain new difficulty estimates that are extracted from the combination of graph colouring heuristics with a heuristic modifier using a linear approach. Different weights are assigned to each parameter and the effect of weights associated with ordering using different heuristics on the quality of the examination schedules is investigated. It is worth noting that the use of information from these heuristics and a heuristic modifier can lead to improvements in the obtained solution. This approach has been tested on two datasets, i.e. Toronto and ITC2007 (The Second International Timetabling Competition) benchmark datasets and has shown to produce high quality solutions that are comparable to other approaches in the literature (Qu et al., 2009).

An overview of adaptive ordering heuristics is presented in Section 2. Section 3 provides the implementation, the instances focused on as part of this study and an analysis of the results. Finally a conclusion is provided in Section 4.

2. An adaptive linear combination of heuristics orderings

An adaptive approach to examination timetabling based on priorities was proposed in Burke and Newall (2004). This approach was extended in Rahman et al. (2009) by introducing additional strategies to improve the solution quality. This involved including methods to choose the ordering of examinations and their assignment to time-slots. The method is based on the idea of squeaky wheel optimisation initiated by Joslin and Clements (1999). Squeaky wheel optimisation is a greedy approach and works by iteratively cycling around three procedures: *Constructor*, *Analyzer* and *Prioritizer*. In relation to the examination timetabling problem, the procedures are as follows:

- *Constructor*. First, the constructor generates an initial solution for a set of unscheduled examinations based on the initial ordering (which can be generated by a chosen graph colouring heuristic). The unscheduled examinations are individually assigned to the best time-slot, i.e. whichever generates the least penalty. During the assignment, there is a possibility that some of the examinations cannot be assigned to a time-slot due to the existence of conflicts with other examinations. In this case, such examinations remain unscheduled.
- *Analyzer*. Once the constructor has completed the assignment, each examination is analysed to check whether there was a problem with the assignment, i.e. whether there is a conflict with other examinations during the assignment. A strategy is used to increase the priority of problematic examinations so that they will be given a higher priority in the next iteration. A certain value is added to the difficulty value