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A semiparametric Bayesian approach to the analysis of financial time series with applications to value at risk estimation

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ABSTRACT

GARCH models are commonly used for describing, estimating and predicting the dynamics of financial returns. Here, we relax the usual parametric distributional assumptions of GARCH models and develop a Bayesian semiparametric approach based on modeling the innovations using the class of scale mixtures of Gaussian distributions with a Dirichlet process prior on the mixing distribution. The proposed specification allows for greater flexibility in capturing the usual patterns observed in financial returns. It is also shown how to undertake Bayesian prediction of the Value at Risk (VaR). The performance of the proposed semiparametric method is illustrated using simulated and real data from the Hang Seng Index (HSI) and Bombay Stock Exchange index (BSE30).

1. Introduction

Financial time series analysis gives practical and theoretical understanding of data collected on financial markets, such as stock and commodity prices, exchange rates or bond yields. Financial data usually consists of a time series of prices of a certain asset for a given period of time. However, most of the financial analysis consider asset returns, which measures the relative changes in prices, as they have more attractive statistical properties. Investors and financial managers need to understand the behavior of asset returns to have good expectations about future returns and the risks to which they will be exposed. Although forecasting is an essential component of any interesting activity, it is usually very difficult to obtain accurate predictions. Since the prediction methods inherently depends on the underlying distributions assumed, it appears to be more appropriate to gain insights into the assumed probability distributions to obtain better predictions for future values. Correctly specifying the distribution is also important as it provides with a measure of investment risk.

Financial market returns exhibit several interesting and complicated features such as volatility clustering and high kurtosis, which make their modeling a challenging task. Engle (1982) introduced the autoregressive conditional heteroskedasticity (ARCH) model to describe these features. Since then, many alternative specifications have been proposed including the stochastic volatility (SV) model (Taylor, 1982), the generalized autoregressive conditional heteroskedasticity (GARCH) model (Bollerslev, 1986), the exponential generalized autoregressive conditional heteroskedasticity (EGARCH) model (Nelson, 1991) and the GJR-GARCH model (Glosten, Jaganathan, & Runkle, 1993), among many others. In order to derive the probability distributions of future returns implied by GARCH-type models, it is necessary to specify the distribution of the innovations. The simplest and most routinely used assumption is that the innovations are Gaussian distributed. However, GARCH models with Gaussian innovations are inconsistent with the excess kurtosis frequently observed in both the conditional and unconditional distributions of returns. Alternative approaches are the Student-t distribution (Bollerslev, 1987), the generalized error distribution (Nelson, 1991), a mixture of two zero mean Gaussian distributions (Bai, Rusell, & Tiao, 2003 and Ausín & Galeano, 2007) or the Pearson's type IV distribution (Bhattacharyya, Chaudhary, & Yadav, 2008).

In this paper, we consider a semiparametric Bayesian approach to the analysis of financial time series. The usual parametric distributional assumptions on the innovations of GARCH models are relaxed by using the class of Gaussian scale mixtures which is a broad class that includes, among others, the Gaussian, Student-*t*, logistic, double exponential, Cauchy and generalized hyperbolic distributions. Then, we further assume that the scale mixing distribution follows a Dirichlet process (DP) prior (Ferguson, 1973), which results in a DP mixture (DPM) model (Antoniak, 1974). DPM models are included in the area referred to as "Bayesian nonparametrics", which actually deal with infinite-dimensional sets of

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parameters, see e.g. Gershman and Blei (2012) for a recent overview. Models of this type allow for greater flexibility in capturing the usual patterns usually observed in financial returns.

Although new to the GARCH framework, DPM models have an extensive literature in Bayesian analysis and provide a broad and flexible class of distributions in many different settings, see, for instance, Ishwaran and Zarepour (2002); Basu and Chib (2003); Damien, Galenko, Popova, and Hanson (2007) and Ghosh, Basu, and Tiwari (2009) and the references therein. In the context of stochastic volatility models, semiparametric Bayesian approaches have been developed recently by Jensen and Maheu (2010) and Delatola and Griffin (2011). These papers have shown, for instance, that estimates of volatility using the semiparametric Bayesian approach can differ dramatically from those using a Gaussian return distribution if there is evidence of a heavy-tailed return distribution.

Extreme price movements in financial markets are unusual, but important. Recently, the large daily price movements have pointed out the need of reliable investment risk measures. Value at Risk (VaR) has become the most widely used measure of market risk, see Jorion (2006). VaR indicates the potential loss associated with an unfavorable movement in market prices over a given time period at a certain confidence level. Statistically speaking, VaR is a quantile of the conditional distribution of the returns. Thus, its calculation strongly depends on the assumptions made for the innovation distribution. Indeed, one of the main criticisms to the use of VaR as a risk measure is the inaccuracy of the VaR estimates produced by standard models, due to the inappropriate specification of the return distribution, see e.g. Kiesel and Kleinow (2002). As the semiparametric approach that we propose is based on a flexible specification of the innovation distribution, it is expected that more accurate VaR estimates can be obtained. In order to illustrate this, we will show how to obtain predictive distributions of future returns which give us the VaR estimates.

The rest of this paper is organized as follows. Section 2 introduces the proposed class of models which shall be referred to as DPM-GARCH models. Section 3 shows how to implement Bayesian inference for these models by developing a Markov Chain Monte Carlo (MCMC) algorithm to sample from the joint posterior distribution. The proposed algorithm combines the ideas of the retrospective sampling proposed in Papaspiliopoulos and Roberts (2008) and the slice sampling approach of Walker (2007). Then, Section 4 explains how to estimate the predictive distribution of the returns and addresses the problem of VaR estimation. Section 5 presents a brief Monte Carlo experiment which illustrates the accuracy in parameter estimation, prediction of returns and VaR estimation. Section 6 analyzes real data from the Hang Seng Index (HSI) and Bombay Stock Exchange index (BSE30), finally, Section 7 concludes.

2. Dirichlet process mixture GARCH-type models

The usual structure of GARCH-type models assumes that a return series, denoted by r_t, can be written as follows:

$$r_t = \mu + h_t^{1/2}\epsilon_t, \tag{1}$$

where μ is the unconditional mean of r_t, which is constant over time, h_t is the conditional variance of r_t given the past history, $F_{t-1} = \{r_{t-1}, r_{t-2}, \ldots\}$, commonly called the conditional volatility, and ϵ_t is a sequence of independent and identically distributed random variables with zero mean and unit variance.

GARCH-type models describe the conditional volatility h_t as an exact function of the past. For instance, the GJR-GARCH (p,q) model proposed by Glosten et al. (1993) assumes that $h_t = \omega + \sum_{i=1}^{p}(\alpha_i + \phi_i I_{t-i})(r_{t-i} - \mu)^2 + \sum_{j=1}^{q}\beta_j h_{t-j}$, where I_{t-i} is equal to 1 if $r_{t-i} < \mu$, and 0 otherwise. Here $\omega > 0$, $\alpha_i \geqslant 0$ and $\alpha_i + \phi_i \geqslant 0$ for $i = 1, \ldots, p$, and $\beta_j \geqslant 0$, for $j = 1, \ldots, q$, to ensure nonnegativity of h_t and $\sum_{i=1}^{p}(\alpha_i + \phi_i/2) + \sum_{j=1}^{q}\beta_j < 1$ to ensure covariance stationarity of r_t. If $\phi_1 = \cdots = \phi_p = 0$, the GJR-GARCH model reduces to the GARCH model. Moreover, if $\beta_1 = \cdots = \beta_q = 0$, the GARCH model reduces to the ARCH model.

As mentioned previously, typical models for the innovation distribution include the Gaussian, Student-t, Gaussian mixture, generalized error or the Pearson's type IV distributions, among others. The aim of this paper is to construct a robust alternative to these usual distributional assumptions. Therefore, we assume that ϵ_t follows an unknown distribution with zero mean and unit variance. However, in general, the unit variance restriction will make it difficult to undertake semiparametric Bayesian inference. Thus, in order to avoid it, we propose rescaling the model defined in Eq. (1) as follows,

$$r_t = \mu + \tilde{h}_t^{1/2}\xi_t, \tag{2}$$

where $\tilde{h}_t = h_t/\omega$ is a rescaled volatility. For instance, in the particular case of the GJR-GARCH model, $\tilde{h}_t$ is given by,

$$\tilde{h}_t = 1 + \sum_{i=1}^{p}(\tilde{\alpha}_i + \tilde{\phi}_i I_{t-i})(r_{t-i} - \mu)^2 + \sum_{j=1}^{q}\beta_j \tilde{h}_{t-j}, \tag{3}$$

where $\tilde{\alpha}_i = \alpha_i/\omega, \tilde{\phi}_i = \phi_i/\omega$ and $\xi_t = \omega^{1/2}\epsilon_t$ is a sequence of independent and identically distributed random variables with zero mean and variance ω.

Now, in order to impose a flexible zero mean distribution on the rescaled innovations, ξ_t, we propose using the broad class of Gaussian scale mixtures, with density function (with respect to Lebesgue measure) given by,

$$f_\xi(\xi_t|G) = \int \phi(\xi_t|0, \lambda_t^{-1})dG(\lambda_t^{-1}), \tag{4}$$

where $\phi(\xi_t|0, \lambda_t^{-1})$ denotes the density function of the Gaussian distribution with zero mean and variance λ_t^{-1}, and G is the scale mixing distribution. The key feature of the proposed semiparametric approach is that we assume that the scale mixing distribution, G, is unknown and modeled by a DP, resulting in a DPM model, which can be written hierarchically as follows,

$$\begin{aligned} &\xi_t|\lambda_t \sim N(0, \lambda_t^{-1}),\\ &\lambda_t|G \sim G,\\ &G|\nu, G_0 \sim DP(\nu, G_0),\\ &\nu \sim \pi(\nu), \end{aligned} \tag{5}$$

where ν is a concentration parameter, with prior density π, and G_0 is the baseline probability measure of the Dirichlet process. Also, the DP provides a conjugate family of priors over distributions such that, given a set of independent draws from G, the posterior distribution is given by,

$$G|\{\lambda_t\}_{t=1}^{T} \sim DP\left(\nu + T, \frac{\nu}{\nu + T}G_0 + \frac{T}{\nu + T}\widehat{G}\right),$$

where $\widehat{G}$ is the empirical distribution for the sample $\{\lambda_t\}_{t=1}^{T}$. In what follows, the GARCH-type models previously given, assuming that ξ_t follows the density function in Eq. (4) and the scale mixing distribution G is modeled by a DP prior as defined above, will be called Dirichlet Process Mixture GARCH-type models, i.e. DPM-ARCH, DPM-GARCH and DPM-GJR-GARCH models, respectively.

It can be shown that, with probability one, G is a discrete distribution with infinite support. Therefore, the scale mixture in Eq. (4) can be interpreted as an infinite Gaussian mixture, which is the alternative representation of the model in Eq. (5) based on the stick-breaking construction of Sethuraman (1994),