European Journal of Operational Research 232 (2014) 405–411

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

A novel optimal preventive maintenance policy for a cold standby system based on semi-Markov theory

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ARTICLE INFO

Article history:
Received 5 April 2012
Accepted 15 July 2013
Available online 27 July 2013

Keywords:
Cold standby
Semi-Markov process
Preventive maintenance
Laplace transform

ABSTRACT

A novel optimal preventive maintenance policy for a cold standby system consisting of two components and a repairman is described herein. The repairman is to be responsible for repairing either failed component and maintaining the working components under certain guidelines. To model the operational process of the system, some reasonable assumptions are made and all times involved in the assumptions are considered to be arbitrary and independent. Under these assumptions, all system states and transition probabilities between them are analyzed based on a semi-Markov theory and a regenerative point technique. Markov renewal equations are constructed with the convolution of the cumulative distribution function of system time in each state and corresponding transition probability. By using the Laplace transform to solve these equations, the mean time from the initial state to system failure is derived. The optimal preventive maintenance policy that will provide the optimal preventive maintenance cycle is identified by maximizing the mean time from the initial state to system failure, and is determined in the form of a theorem. Finally, a numerical example and simulation experiments are shown which validated the effectiveness of the policy.

1. Introduction

When large-scale complex systems fail, the consequence on manufacturing and human safety can be devastating, such as when plants are shut down, chemical plants explode, or airplanes crash. However, due to site environments and economic reasons, it is almost impossible for practical engineering systems to realize zero risk and/or be completely safe. Therefore, knowing how to improve the system reliability and safety effectively is important. Consequently, improving preventive maintenance has become an efficient and objective approach to reducing risks.

Recently, the optimization of maintenance policies for repairable systems has attracted the attention of many researchers due to their various applications in engineering. From a methodology point of view, most optimal maintenance policies focus on modeling the repairable system and optimizing a certain performance indicator (e.g., maintenance cost, system availability, or reliability) under some constraints. For instance, Smidt-Destombes, Heijden, and Harten (2009) developed heuristics for a joint optimization of preventive maintenance (PM) frequency, spare part inventory levels and spare part repair capacity for a k-out-of-n system. Wang (2012), whose work is similar to Smidt–Destombes, proposed a joint optimization for inventory control of spare parts and the PM inspection interval, in which stochastic dynamic programming was employed to find the joint optimal solutions over a finite time horizon. A study by Dehayem Nodem, Kenné, and Gharbi (2011) which aimed to find the decision variables that minimize overall cost, including repair and PM costs, resulted in a method which found the optimal production, repair, and PM policies based on a semi-Markov decision process. Borrero and Akhavan-Tabatabaei (2013) formulated two analytical models based on MDPs for a single-machine, single-produce workstation subject to random failure; the purpose of these models was to obtain optimal policies using a cost function associated with three types of cost. More information on optimal maintenance strategies can be found in studies by Yeh and Lo (2001), Kyriakidis and Dimitrakos (2006), Bedford, Dewan, Meilijson, and Zitrou (2011), and Fallah-Fini, Triantis, Garza, and Seaver (2012).

From the volume of published literature, it is clear that the semi-Markov Process (SMP) is commonly used to model repairable systems in many maintenance policies (Çekyay & Özekici, 2010; Burak & Sloan, 2013), which shows that the SMP can characterize system dynamics and facilitate modeling a variety of systems well. On the other hand, many authors view the costs associated with maintenance, repair and replacement as a key factor in their work (Smith & Dekker, 1997; Jia & Wu, 2009; Zhang & Wang, 2011). However, for certain complex systems, such as space exploration

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http://dx.doi.org/10.1016/j.ejor.2013.07.020

Nomenclature

C_i	component i $i = 1, 2$
$F(t)$	the cumulative distribution function (CDF) of component lifetime
$G_1(t)$	the CDF of repair time for failed component
$G_2(t)$	the CDF of maintenance time
$1/\lambda$	the failure rate of components
μ_1	the repair rate, the mean of $G_1(t)$
μ_2	the maintenance rate, the mean of $G_2(t)$
X	the random variable denoting component lifetime
Y_1	the random variable denoting repair time
Y_2	the random variable denoting maintenance time
T	the PM cycle
T^o	the optimal PM cycle
X_T	the random variable denoting T
$H(t)$	the CDF of X_T
$p_{ij}(t)$	the CDF for system transits from regenerative state S_i to S_j
$\pi_i(t)$	the CDF of time from entering S_i to system failure
m_i	the mean of $\pi_i(t)$
E	the states space $E = \{S_i \mid i = 0, 1, 2, 3\}$
*	the convolution operator
^	the symbol denoting the result of Laplace transform for a variable

and satellite systems, or complex control systems in steel or chemical plants, system reliability is much more important than upkeep costs. Under such circumstances, effort made to minimize upkeep costs may not be as relevant. Hence, situations where costs are a secondary factor are not taken into account in our study.

In addition to maintenance policies, the structure of a system has a significant effect on the system availability and reliability. Standby redundancy, as one of the important structures in reliability theory, attracts a substantial amount of interest in the field of system reliability and operational research. The common standby structures include the 2-or 3-unit standby (Subramanian & Anantharaman, 1995;Mahmoud & Moshref, 2010), the $(2, n-2)$ standby (Papageorgiou & Kokolakis, 2007) and (Papageorgiou & Kokolakis, 2010), the k-out-of-N structure (Ding, Zuo, Tian, & Li, 2010;Ruiz-Castro & Li, 2011; Smidt-Destombes et al., 2009;Li & Zuo, 2008) and the M-for-N shared protection structure (Hirokazu & Atsushi, 2011). Among such structures, the 2-unit/component cold standby structure studied in this paper is a practical aspect of standby redundancy, and has been widely applied in engineering design, such as in space exploration and satellite systems (Sinaki, 1994), textile manufacturing systems (Pandey, Jacob, & Yadav, 1996), carbon recovery systems in fertilizer plants (Kumar, Kumar, & Mehta, 1996). It is worth mentioning that the issue studied in this paper comes from the practical flattener control system in a steel plant and the optimal results obtained have been applied to the PM policy of the flattener control system.

In view of the above discussion, in this paper, a novel optimal PM policy for a cold standby system with two components and a repairman is presented. In this study, a SMP and regenerative point technique was used to model the system under some reasonable assumptions and all possible states of the system and transition probabilities between them were analyzed. The optimal PM cycle was derived by maximizing the mean time from the initial state to system failure in the form of the theorem. Finally, results from a numerical example and simulation experiments, which were used to validate the effectiveness of the optimal PM policy are provided.

The remainder of this paper is organized as follows: in Section 2 the basic assumptions for system modeling are given, and all possible states of the system are determined. In Section 3, the semi-Markov kernel of the system corresponding to transition probabilities between states are further analyzed. In Section 4, the optimal PM cycle is derived by maximizing the mean time from the initial state to system failure. In Sections 5 and 6 a numerical example and simulation experiment is performed, and a conclusion with a brief summary is provided. Finally, in, Appendix A, the Proof of theorem 1 is given.

For ease of reference, some notations to be used in this paper are given as follows:

2. Modeling description and assumptions

The system under consideration consists of a working component and a cold-standby spare component. It should be noted that the working and spare components are not different from each other, meaning that the two components have the same distribution of time-to-failure. In addition, the system includes a repairman who is responsible for the repair of the failed component and the PM of the working one. For simplicity, the following reasonable and necessary assumptions of the system were made.

Assumption 1. Two components are the same type so that their lifetimes are independent and identically distributed (i.i.d.). After repair or maintenance, both components are considered to be "as good as new". Repair and maintenance times are arbitrary and independent. For simplicity of expression, one component is called C_1 and the other is called C_2.

Assumption 2. Initially, the two components are both new and it is supposed that C_1 is in a working state and C_2 is in a cold standby state. If C_1 fails, C_2 begins to work instead of C_1 instantaneously. At this same time, the repairman begins repairing C_1. The time to switch from C_1 to C_2 is negligible or is not taken into account.

Assumption 3. The failed component, after having been repaired, either begins to work or enters the cold standby state according to the state of the other component.

Assumption 4. In order to keep components' running properly, the working component needs to receive PM periodically at times $kT(k = 1, 2, \ldots, n)$ where T is a cycle length, provided that the spare component is available. If the spare component is not available, the PM is skipped until the next time for it to be done.

Assumption 5. When the working component fails while the other is still being repaired or maintained, the system fails.

Under above assumptions, the various state of the system can be straightforwardly defined as follows:

S_0	A component works while the other is in cold standby. Both components are new at the time of entry into this state.
S_1	A component works while the other is in repair.
S_2	A component works while the other is in maintenance.
S_3	A component is in repair or maintenance while the other waits for repair or maintenance, this state represents system failure.